Calculus & Analytic Geometry III

Week 3 HW—Taylor Series

These problems use the techniques of TN §5. Each problem can be derived from the basic series given in Examples 4.2. **Hand in your thoughtful, well-written, and reflective solution to problem 10 on Thursday, April 17.**

Directions
(a) Find the Taylor series for $f(x)$ based at b. Your answer should have one Sigma ($\sum$) sign. On some problems you might want to describe the coefficients using a multi-part notation as in Example 5.5.

(b) Then write the solution in expanded form: $a_0 + a_1(x-b) + a_2(x-b)^2 + \ldots$ where you write at least the first three non-zero terms explicitly.

(c) Then give an interval I where the Taylor series converges.

1. $f(x) = \cos(3x^2)$ based at $b = 0$.
2. $f(x) = \sin^2(x)$ based at $b = 0$.
3. $f(x) = e^{4x-5}$ based at $b = 2$.
4. $f(x) = \sin(x)$ based at $b = \pi/2$.
5. $f(x) = \dfrac{1}{4x-5} - \dfrac{1}{3x-2}$ based at $b = 0$.
6. $f(x) = \dfrac{x}{(2x+1)(3x-1)}$ based at $b = 1$.
7. The "sinh" and "cosh" functions are used, for example, in electrical engineering, and are defined by $\sinh(x) = (e^x - e^{-x})/2$, and $\cosh(x) = (e^x + e^{-x})/2$. Do questions (a) and (b) above for the function $h(x) = 2\sinh(3x) - 4\cosh(3x)$ based at $b = 0$.
8. $f(x) = \dfrac{1}{(2x-5)^2(3x-1)}$ based at $b = 0$.
9. $f(x) = \ln(1+x^2)$ based at $b = 0$.
10. $f(x) = \dfrac{2x}{1+x^2}$ based at $b = 0$.

    Do this problem in two ways: (a) Find the series for $1/(1+x^2)$ then multiply it by $2x$ and (b) Differentiate $\ln(1+x^2)$ and use problem (9). Your answer to (b) should agree with your answer to (a).
11. Find the fifth Taylor polynomial based at $b = 0$ for $f(x) = e^x \sin x$ by multiplication of the series for e^x and $\sin x$ (you do not have to find the general term of the product).