# From the Desk of

***Tuft Hunter***

April 8, 2008

Independent Mathematical Contractors, Inc.
CP 108
Tacoma, WA 98402

Dear IMC:

Please forgive the intrusion, but I have an unusual request. I am the personal assistant to an extremely rich and rather eccentric recluse. Since reading Dan Brown's novel *The DaVinci Code* and more recently, seeing the movie *Mr. Magorium's Magic Emporium*, he has become obsessed with the Fibonacci Numbers.

He scoured sources far and wide to learn everything that he can. Having made his money in aviation, he only puts stock in mathematics that can be explained *in the language of calculus*. This is where you come in. My calculus background is limited but I want to impress my employer by presenting him a derivation of Binet's formula based on power series and partial fraction decompositions. I have secured a document with an outline of the derivation, but need you to teach me the details. My hope is to pander to Mr. H's obsession, working my way into his inner circle and eventually inheriting a small fortune.

To assure our success in this collaboration, Dr. Jennifer Quinn, has agreed to field any technical questions you might have in the course of your investigation. She is an expert on combinatorial interpretations of Fibonacci Numbers though recommended you for your calculus expertise. You should plan to see her by **April 18$^{st}$** with an indication of your working group and preliminary progress on the project.

I look forward to receiving your report, which should be typewritten and 3–5 pages in length. As Mr. H. may quiz me on my knowledge, I will be expected to justify every state that I make. Please do the same. You may to submit your report electronically, but pdf documents are strongly preferred. Respond by midnight, **May 2**.

Sincerely,

*Tuft Hunter*

Tuft Hunter
Personal Assistant

# Fibonacci Numbers

You may have heard about the Fibonacci sequence of numbers. It starts with the numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, · · ·

Even if you haven't seen them before you may recognize the pattern. To get each of the numbers after the second one you simply add the two previous numbers. This gives an easy way to write out the Fibonacci sequence as far as you wish. Suppose that you only wanted to know the $100^{th}$ term of the sequence and you did not necessarily want to know all the numbers that came before it. Is there a formula which would give you this number directly? In this project you will discover the formula. Follow the steps below:

a. Let $a_1, a_2, a_3, \ldots$ represent the Fibonacci sequence. Consider the power series $f(x) = a_1 x + a_2 x^2 + a_3 x^3 + \cdots$. Show that the radius of convergence of this power series is at least 1/2
b. Find a polynomial *p(x)* such that *p(x)f(x) = x*.
c. Find the power series expansion for $\frac{1}{ax+b}$ and determine its radius of convergence.
d. Find the partial fraction decomposition for $\frac{x}{p(x)}$ and use it to determine the coefficients for *f(x).*
e. What is the radius of convergence for *f(x)*?
f. Use your result from d) to determine a formula for $a_k$ for each value of *k*. Verify that the formula you derived is correct for small values of *k*. Then determine the value of $a_{100}$ and a few other values of $a_k$ for *k* large.

Adapted from Neal Brand, University of North Texas, www.math.unt.edu/~brand/projects/calc2/calc2.htm