Quadratic Equations in 3-D

A **quadric surface** is the graph of a second-degree equation in three variable x, y, and z.

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

where $A, B, \ldots, J$ are constants (and at least one of A, B, or C is not zero).

We have already seen the equation of a sphere...

What happens when one of the variables is missing?

$x^2 + z^2 = 4$ $\qquad$ $x - y^2 = 1$ $\qquad$ $\frac{y^2}{4} + \frac{z^2}{9} = 1$

These are called **cylinders**. *The parallel lines are called* **rulings**.

Quick Review of Conics Sections from §10.5:

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[1] The equation of a *parabola* with focus $(0, p)$ and directrix $y = -p$ is

$$x^2 = 4py.$$

The equation of a parabola with focus $(p, 0)$ and directrix $x = -p$ is

$$y^2 = 4px.$$

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[4] The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a \geq b > 0$$

has foci $(\pm(a^2 - b^2), 0)$ and vertices $(\pm a, 0)$, $(\pm b, 0)$.

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[7] The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad a \geq b > 0$$

has foci $(\pm(a^2 + b^2), 0)$ and vertices $(\pm a, 0)$ and asymptotes $y = \pm(b/a)x$.

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Extension to Three-Dimensions

1. $9x^2 + 36y^2 + 4z^2 = 36$
2. $4x^2 + 9y^2 - 4z^2 = 0$
3. $36x^2 + 9y^2 - 4z^2 = 36$
4. $4x^2 - 9y^2 - 4z^2 = 36$
5. $9x^2 + 4y^2 - 6z = 0$
6. $9x^2 - 4y^2 - 6z = 0$
7. $4x^2 + y^2 + 4z^2 - 4y - 4z + 36 = 0$

ellipsoid • *cone* • *hyperboloid of one sheet* • *hyperboloid of two sheets* • *elliptic paraboloid* • *hyperbolic paraboloid*