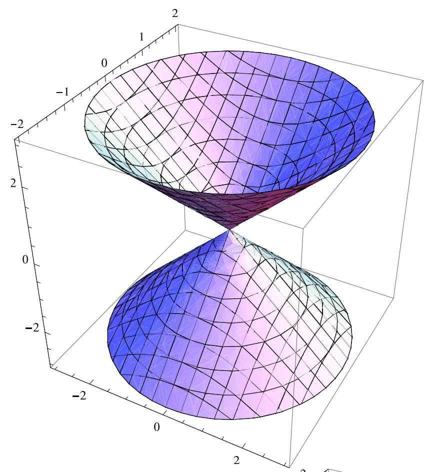
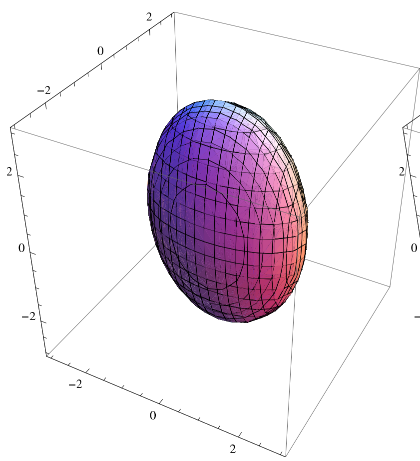
Calculus & Analytic Geometry III

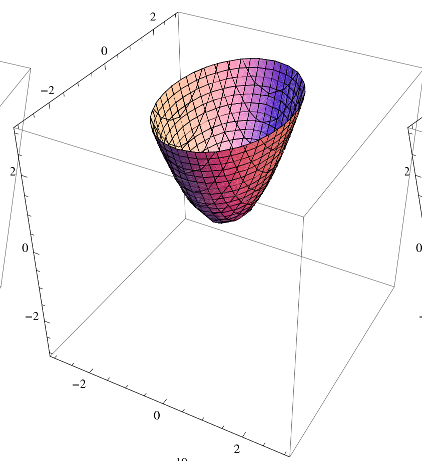
Quadratic Equations in 3-D

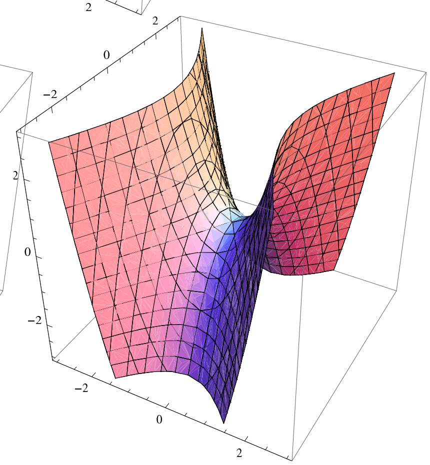
Match each function to its graph

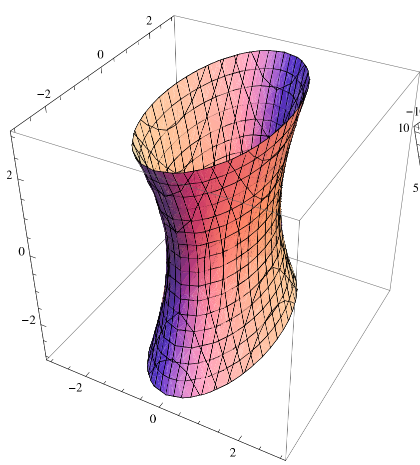
1. $9x^2 + 36y^2 + 4z^2 = 36$
2. $4x^2 + 9y^2 - 4z^2 = 0$
3. $36x^2 + 9y^2 - 4z^2 = 36$
4. $4x^2 - 9y^2 - 4z^2 = 36$
5. $9x^2 + 4y^2 - 6z = 0$
6. $9x^2 - 4y^2 - 6z = 0$
7. $4x^2 + y^2 + 4z^2 - 4y - 4z + 36 = 0$
8. $4x^2 + y^2 + 4z^2 - 4y - 4z - 36 = 0$

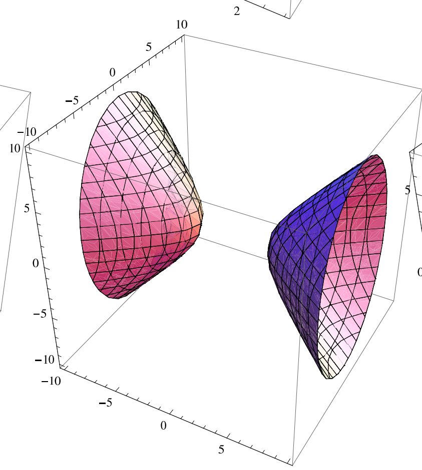


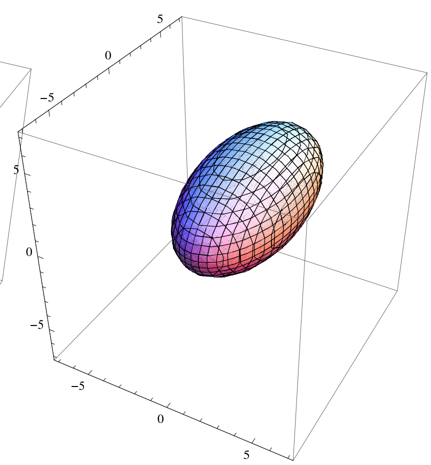












cone • *ellipsoid* • *elliptic paraboloid* • *hyperbolic paraboloid* • *hyperboloid of one sheet* • *hyperboloid of two sheets*

Calculus & Analytic Geometry III

Parametric Equations (§10.1) and Vector Functions (§13.1)

Definition. If x and y are given as continuous function

$$x = f(t) \qquad\qquad y = g(t)$$

over an interval of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equation is a *parametric curve* (sometimes called a*plane curve*). The equations are *parametric equations* for the curve.

Often we think of parametric curves as describing the movement of a particle in a plane over time.

Examples.

$$\left.\begin{array}{rcl} x &=& 2\cos t \\ y &=& 3\sin t \end{array}\right\} \quad 0 \le t \le \pi \qquad\qquad \left.\begin{array}{rcl} x &=& e^t \\ y &=& \ln t \end{array}\right\} \quad 1 \le t \le e$$

Can we find parameterizations of known curves?

the line segment from $(1, 3)$ to $(5, 1)$

circle $x^2 + y^2 = 1$

Why restrict ourselves to only moving through planes? Why not space? And why not use our nifty vector notation?

Definition. If x, y, and z are given as continuous functions

$$x = f(t) \qquad y = g(t) \qquad z = h(t)$$

over an interval of t-values, then the set of points $(x, y, z) = (f(t), g(t), h(t))$ defined by these equation is a *parametric curve* (sometimes called a *space curve*). The equations are *parametric equations* for the curve.

Examples.

$$\left.\begin{aligned} x &= 2\cos t \\ y &= 3\sin t \\ z &= t \end{aligned}\right\} \quad 0 \le t \le \pi \qquad\qquad \left.\begin{aligned} x &= 5 + t \\ y &= 1 + 4t \\ z &= 3 - 2t \end{aligned}\right\} \quad 0 \le t \le 1$$

An oval helix

The line in 3-D

$\mathbf{r}(t) = (5+t)\mathbf{i} + (1+4t)\mathbf{j} + (3-2t)\mathbf{k}$

from §12.5

The *vector function* $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is the position vector of the point $P(f(t), g(t), h(t))$on the space curve C. The tip of the moving vector $\mathbf{r}$ traces out the space curve—provided everything is continuous.

Questions.

1. Do vector functions make sense with our original definition of function as a rule that assigns one output for each input?

2. What should the domain of a vector function be?

State the natural domains for:

$\mathbf{r}(t) = \langle t^2, t^3, \sqrt{t} \rangle$

$\mathbf{r}(t) = \langle \sin^{-1}(t), \ln(t), 1 \rangle$

3. What should it mean for a vector function to have a limit at a point a?

Find the limits:

$\lim_{t \to 0} \langle t^2, t^3, \frac{\sin t}{t} \rangle$

$\lim_{t \to 0^+} \langle \sin^{-1}(t), \ln(t), 1 \rangle$

4. What should it mean for a vector function to be continuous at a point a?

5. Can you see that we are leading to asking calculus questions about vector functions?

Describe the graphs for

$\mathbf{r}(t) = \langle \cos(4t), t, \sin 4t \rangle$

$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \sin 5t\mathbf{k}$