# Calculus & Analytic Geometry III

## Motion in Space: Velocity and Acceleration

**Warm-up.** If the position of an object is given by $s(t) = \cos(\pi t/4)$ for $0 \leq t \leq 10$. Find its velocity, speed, and acceleration at time $t$.

**Moving through space** works the same way except position, velocity, and acceleration become vector quantities.

| Motion in a plane | Motion through space |
|---|---|
| $s(t)$ is position | $\mathbf{r}(t)$ is position |
| $s'(t) = v(t)$ is velocity | $\mathbf{r}'(t) = \mathbf{v}(t)$ is velocity |
| $s''(t) = v'(t) = a(t)$ is acceleration | $\mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t)$ is acceleration |

**Example** Find the velocity, speed and acceleration of a particle with the given position function.

$\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j} + 2\mathbf{k}$ $\qquad$ $\mathbf{r}_2(t) = t\sin t\mathbf{i} + t\cos t\mathbf{j} + t^2\mathbf{k}$

Of course, we can use integrals of vector functions to work backwards and find a position vector given information about velocity (or acceleration. )

**Problem.** Find the velocity and position vectors for a particle with acceleration $\mathbf{a}(t) = 2\mathbf{i} + 6t\mathbf{j} + 12t^2\mathbf{k}$ provided $\mathbf{v}(0) = \mathbf{i}$ and $\mathbf{r}(0) = \mathbf{j} - \mathbf{k}$.

All of mechanics can be filtered through the lens of vector functions. $\mathbf{F}(t) = m\mathbf{a}(t)$.

**Problems.**

1. What force is required so that a particle of mass $m$ has the position function $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$.?

2. A force with magnitude 20 N acts directly upward from the $xy$-plane on an object with mass 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and its speed at time $A$ projectile is fired with an initial speed of 500 m/s and angle of elevation 30°. Find (a) the range of the projectile, (b) the maximum height reached, and (c) the speed at impact.

Sometimes it is useful to resolve the acceleration vector into two components

- one in the direction of motion (i.e. in the direction of the tangent vector)
- one in the direction of the normal.

Recall that $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} =$

So $\mathbf{v} =$ Now differentiate with respect to $t$.

This leads to $$\boxed{\mathbf{a} = v'\mathbf{T} + \kappa v^2\mathbf{N}.}$$

**Example.** Find the tangential and normal components of the acceleration vector $\mathbf{r}(t) = e^t\mathbf{i} + \sqrt{2}t\mathbf{j} + e^{-t}\mathbf{k}$.