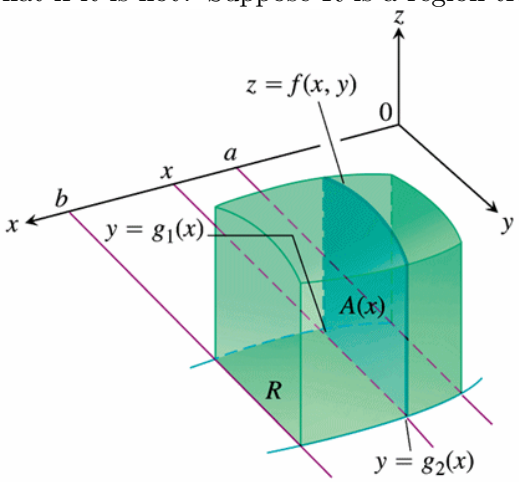
Calculus & Analytic Geometry III

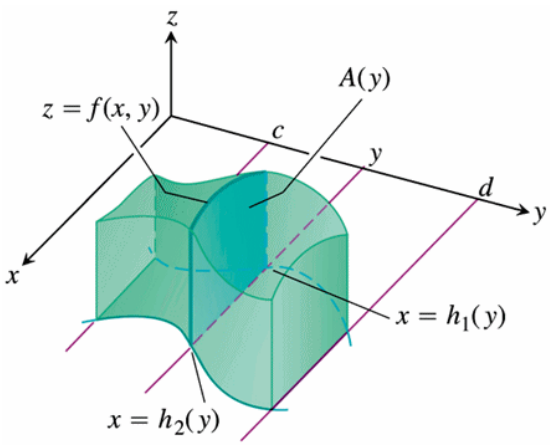
Double Integrals over General Regions

Warm-up. A building is 8 meters wide and 16 meters long. It has a flat roof that is 12 meters high at one corner and 10 meters high at each of the adjacent corners. What is the volume of the building?

$z = 12 - \frac{1}{4}x - \frac{1}{8}y$.
1280 cubic meters

An important aspect of this problem was that the region we were looking over was a rectangle. What if it is not? Suppose R is a region trapped between two curves:





To calculate the area of the vertical slice, $A(x)$:

$$A(x) = \int_{\square}^{\square} f(x,y)dy$$

Then sum the vertical slices as x goes from a to b:

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dy\,dx$$

To calculate the area of the vertical slice, $A(y)$:

$$A(y) = \int_{\square}^{\square} f(x,y)dx$$

Then sum the vertical slices as y goes from c to d:

$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(x)}^{h_2(x)} f(x,y)dx\,dy$$

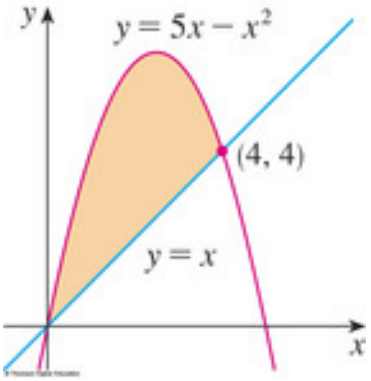
Examples.

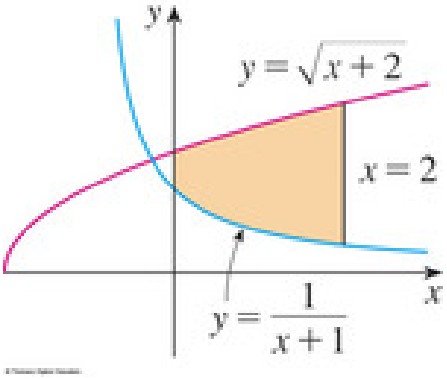
1. Calculate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the xy-plane bounded by the x axis, the line $y = x$ and the line $x = 1$.

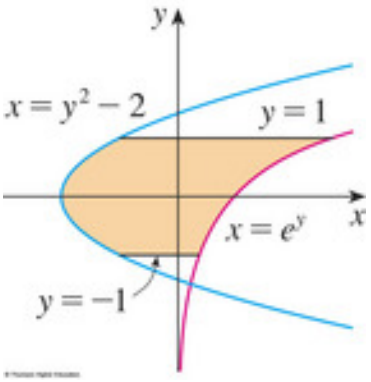
Ans: $1 - \cos 1 \approx .46$

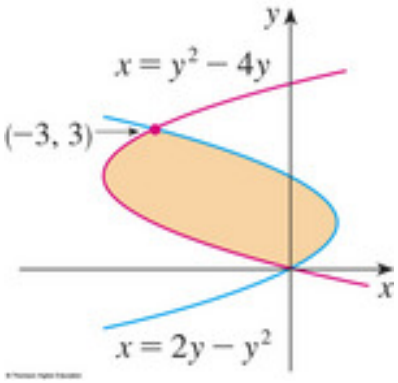
2. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2) dy\, dx$ and write an equivalent integral with the order of integration reversed.

3. For each of the (familiar) regions sketched below, create a double integral to calculate the signed volume of the 3-dimensional solid region over the region R in the xy-plane and the surface $f(x, y) = yx$.









4. As time permits...calculate the double integrals (you may have to reverse the order of integration to evaluate.

$$\int_0^1 \int_y^1 x^2 e^{xy} dx\, dy \qquad \int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy\, dx \qquad \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x dy\, dx$$