CALCULUS & ANALYTIC GEOMETRY III

Review

1. Sequences, series, power series: limits, series of important functions, convergence

2. Taylor polynomials and series

    (a) first Taylor polynomial=linear approximation
    (b) second Taylor polynomial = quadratic approximation
    (c) Taylor polynomials of higher degree
    (d) Error estimates of polynomials (a.k.a Taylor's Inequality)
    (e) applications

3. Vectors

    (a) Cartesian coordinates, distance
    (b) vector—addition, scalar multiplication, magnitude, dot product, cross product, and their properties
    (c) unit vector
    (d) projections
    (e) equations of lines and planes (parametric, symmetric, and vector forms)
    (f) cylinders and quadric surfaces

4. Vector functions

    (a) parametric curves in 2-dimensions: "chain rule" to find slope, area, arc length, surface area
    (b) parametric curves in 3-dimensions (a.k.a. space curves)
        i. derivative, tangent vector, unit tangent vector, integral
        ii. arc length
        iii. curvature
        iv. normal and binormal vectors

5. Functions with two or more inputs

    (a) domain, range, graph
    (b) level curves, contour lines
    (c) partial derivative
    (d) directional derivative, gradient vector
    (e) tangent plane = linear approximation
    (f) finding extremes on closed and bounded regions including second derivative test
    (g) finding extremes subject a constraint (Lagrange)
    (h) multiple integrals (double and triple)
        i. finding area and volumes over general regions
        ii. reversing order of integration
        iii. finding mass, center of mass, and moments (first and second) for lamina with variable density

Questions to guide your review.

1. What is a convergent sequence? What is a convergent series? What does $\lim_{n\to\infty} a_n = 3$ mean? What does $\sum_{n=1}^{\infty} a_n = 3$ mean?

2. What is a geometric series? Under what circumstances is it convergent? What is its sum?

3. What is a p-series? Under what circumstances is it convergent?

4. Write the general form of a power series. What is the radius of convergence of the a power series? What is the interval of convergence of a power series?

5. Write an expression for the n-th degree Taylor polynomial of f centered at a. What is the difference between this *polynomial* and the Taylor *series* of f centered at a?

6. What is Taylor's Inequality? How is it frequently applied?

7. What are the Taylor series about about 0 and the intervals of convergence for (a) $\frac{1}{1-x}$ (b) e^x (c) $\sin x$ (d) $\tan^{-1} x$. (This list is by no means exhaustive....)

8. What are some of the applications of Taylor polynomials?

9. What is the difference between a vector and a scalar?

10. How do you add two vectors geometrically? How do you add them algebraically?

11. How do you find the dot product $\mathbf{a} \cdot \mathbf{b}$ of two vectors if you know their lengths and the angle between them? What if you know their components?

12. How do you find the cross product $\mathbf{a} \times \mathbf{b}$ of two vectors if you know their lengths and the angle between them? What is you know their components?

13. How do you find a vector perpendicular to a plane?

14. How do you find the angle between two intersecting planes?

15. How do you tell if two vectors are parallel? perpendicular?

16. How do you find the area under a parametric curve? the length of a parametric curve? the surface area obtained by revolving a parametric curve?

17. What is a vector function? How do you find its derivative and its integral?

18. What is the definition of curvature?

19. How do you find the velocity, speed, and acceleration of a particle that moves along a space curve?

20. How do you interpret $f_x(x, y)$ and $f_y(x, y)$ geometrically? If $f(x, y)$ is given, how do you calculate f_x and f_y?

21. What does Clairaut's Theorem say?

22. If f is differentiable, write and expression for the directional derivative of f at (x_0, y_0) in the direction of a unit vector $\mathbf{u}$.

23. Define the gradient of f for a function of two or three variables. Explain the geometric significance of the gradient.

24. What does it mean for f to have a local maximum at (a, b)? absolute maximum? saddle point? local minimum? absolute minimum?

25. State the extreme value theorem for functions of two variable. How do you find the values that the Extreme Value Theorem guarantees?

26. Explain how the method of Lagrange multipliers works in find the extreme values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$.

27. How do you evaluate $\iint_R f(x, y) dA$?

28. If a lamina occupied a plane region D and has density function $\rho(x, y)$ write expression for each of the following in terms of double integrals: the mass, the moments about the x- and y-axes, the center of mass, the moments of inertia about the x- and y-axes.