

1. Find the Taylor series for $f(x) = e^x$, based at $b = 2$. Give your answer using summation notation. (/10)

2. Let $f(x) = \frac{1}{(1-x)(1+2x)}$. (/30)

(a) Find the Taylor series for $f(x)$ based at $b = 0$ and the interval of convergence. Give your answer using the summation notation.

(b) Find $f^{(6)}(0)$.

(Hint: I do not expect you to take the derivative 6 times. Instead think about working backwards from the series you found above.)

3. Let $f(x) = 2 \cos(2x) - 1$. (/20)

(a) Find the quadratic approximation $T_2(x)$ of $f(x)$ based at $b = 0$.

(b) Using Taylor's inequality, find a bound on the error $|f(x) - T_2(x)|$ on the interval $[-.1, .1]$.

4. Find the values of x such that the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal. (/10)

5. Let $\mathbf{a} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{j} + 3\mathbf{k}$. Calculate the following quantities: (/20)

(a) $|\mathbf{a}|$

(b) $\mathbf{a} \times \mathbf{b}$

(c) $\text{proj}_{\mathbf{a}} \mathbf{b}$

(d) the angle between \mathbf{a} and \mathbf{b}

6. **Essay Question.** You may answer the question posed for full credit (10 pts) or propose your own essay question and answer for partial credit (8 pts max). If you choose the second option, be sure to clearly state your *well-posed* question. You are to write a paragraph presenting your answer. You may wish to illustrate your discussion with graphs or other diagrams. (/10)

Taylor polynomials are finite approximations of Taylor series. Finding Taylor polynomials only requires the ability to take derivatives. Discuss the advantages and disadvantages of waiting to study Taylor series until the third quarter of Calculus.

* **Bonus Question.** Explain one mathematical topic that you studied to prepare for this examination but feel you did not get the opportunity to adequately show your knowledge. (In other words, *WOW* me with some of your mathematical knowledge.) (/5)