1. $x = t\sin t,\ y = t^2 + t \quad\Rightarrow\quad \dfrac{dy}{dt} = 2t+1,\ \dfrac{dx}{dt} = t\cos t + \sin t$, and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t+1}{t\cos t + \sin t}$.

5. $x = e^{\sqrt{t}},\ y = t - \ln t^2;\ t = 1.\quad \dfrac{dy}{dt} = 1 - \dfrac{2t}{t^2} = 1 - \dfrac{2}{t},\ \dfrac{dx}{dt} = \dfrac{e^{\sqrt{t}}}{2\sqrt{t}}$, and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1-2/t}{e^{\sqrt{t}}/(2\sqrt{t})}\cdot\dfrac{2t}{2t} = \dfrac{2t-4}{\sqrt{t}e^{\sqrt{t}}}$.

When $t = 1$, $(x, y) = (e, 1)$ and $\dfrac{dy}{dx} = -\dfrac{2}{e}$, so an equation of the tangent line is $y - 1 = -\dfrac{2}{e}(x - e)$, or $y = -\dfrac{2}{e}x + 3$.

8. (a) $x = \tan\theta,\ \ y = \sec\theta;\ (1, \sqrt{2}).\quad \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{\sec\theta\tan\theta}{\sec^2\theta} = \dfrac{\tan\theta}{\sec\theta} = \sin\theta.$

When $(x, y) = (1, \sqrt{2})$, $\theta = \frac{\pi}{4}$ (or $\frac{\pi}{4} + 2\pi n$ for some integer n), so $dy/dx = \sin\frac{\pi}{4} = \sqrt{2}/2$.

Thus, an equation of the tangent to the curve is $y - \sqrt{2} = (\sqrt{2}/2)(x - 1)$, or $y = (\sqrt{2}/2)\,x + (\sqrt{2}/2)$.

(b) $\tan^2\theta + 1 = \sec^2\theta \quad\Rightarrow\quad x^2 + 1 = y^2$, so $\dfrac{d}{dx}(x^2+1) = \dfrac{d}{dx}(y^2) \quad\Rightarrow\quad 2x = 2y\,\dfrac{dy}{dx}$. When $(x, y) = (1, \sqrt{2})$,

$\dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$, so an equation of the tangent is $y - \sqrt{2} = (\sqrt{2}/2)(x - 1)$, as in part (a).

13. $x = t - e^t,\ \ y = t + e^{-t} \quad\Rightarrow$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-e^{-t}}{1-e^t} = \frac{1-\dfrac{1}{e^t}}{1-e^t} = \frac{\dfrac{e^t-1}{e^t}}{1-e^t} = -e^{-t} \quad\Rightarrow\quad \frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{dx/dt} = \frac{\dfrac{d}{dt}(-e^{-t})}{dx/dt} = \frac{e^{-t}}{1-e^t}.$$

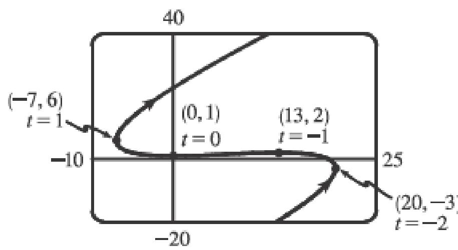
The curve is CU when $e^t < 1$ [since $e^{-t} > 0$] $\quad\Rightarrow\quad t < 0$.

15. $x = 2\sin t,\ y = 3\cos t,\ 0 < t < 2\pi$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3\sin t}{2\cos t} = -\frac{3}{2}\tan t, \text{ so } \frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{dx/dt} = \frac{-\frac{3}{2}\sec^2 t}{2\cos t} = -\frac{3}{4}\sec^3 t.$$

The curve is CU when $\sec^3 t < 0 \quad\Rightarrow\quad \sec t < 0 \quad\Rightarrow\quad \cos t < 0 \quad\Rightarrow\quad \frac{\pi}{2} < t < \frac{3\pi}{2}$.

18. $x = 2t^3 + 3t^2 - 12t,\ y = 2t^3 + 3t^2 + 1$.



$\dfrac{dy}{dt} = 6t^2 + 6t = 6t(t+1)$, so $\dfrac{dy}{dt} = 0 \quad\Leftrightarrow$

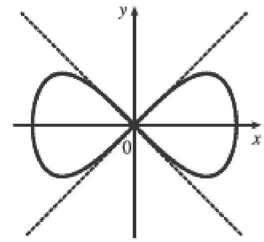
$t = 0$ or $-1 \quad\Leftrightarrow\quad (x, y) = (0, 1)$ or $(13, 2)$.

$\dfrac{dx}{dt} = 6t^2 + 6t - 12 = 6(t+2)(t-1)$, so $\dfrac{dx}{dt} = 0 \quad\Leftrightarrow$

$t = -2$ or $1 \quad\Leftrightarrow\quad (x, y) = (20, -3)$ or $(-7, 6)$.

The curve has horizontal tangents at $(0, 1)$ and $(13, 2)$, and vertical tangents at $(20, -3)$ and $(-7, 6)$.

25. $x = \cos t$, $y = \sin t \cos t$. $dx/dt = -\sin t$, $dy/dt = -\sin^2 t + \cos^2 t = \cos 2t$. $(x, y) = (0, 0) \Leftrightarrow \cos t = 0 \Leftrightarrow t$ is an odd multiple of $\frac{\pi}{2}$. When $t = \frac{\pi}{2}$, $dx/dt = -1$ and $dy/dt = -1$, so $dy/dx = 1$. When $t = \frac{3\pi}{2}$, $dx/dt = 1$ and $dy/dt = -1$. So $dy/dx = -1$. Thus, $y = x$ and $y = -x$ are both tangent to the curve at $(0, 0)$.



29. $x = 2t^3$, $y = 1 + 4t - t^2 \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{4 - 2t}{6t^2}$. Now solve $\dfrac{dy}{dx} = 1 \Leftrightarrow \dfrac{4 - 2t}{6t^2} = 1 \Leftrightarrow$ $6t^2 + 2t - 4 = 0 \Leftrightarrow 2(3t - 2)(t + 1) = 0 \Leftrightarrow t = \frac{2}{3}$ or $t = -1$. If $t = \frac{2}{3}$, the point is $\left(\frac{16}{27}, \frac{29}{9}\right)$, and if $t = -1$, the point is $(-2, -4)$.