

2. (a) From Definition 1, a convergent sequence is a sequence for which $\lim_{n \rightarrow \infty} a_n$ exists. Examples: $\{1/n\}$, $\{1/2^n\}$

(b) A divergent sequence is a sequence for which $\lim_{n \rightarrow \infty} a_n$ does not exist. Examples: $\{n\}$, $\{\sin n\}$

5. $a_n = \frac{3(-1)^n}{n!}$, so the sequence is $\left\{ \frac{-3}{1}, \frac{3}{2}, \frac{-3}{6}, \frac{3}{24}, \frac{-3}{120}, \dots \right\} = \left\{ -3, \frac{3}{2}, -\frac{1}{2}, \frac{1}{8}, -\frac{1}{40}, \dots \right\}$.

6. $a_n = 2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)$, so the sequence is $\{2, 2 \cdot 4, 2 \cdot 4 \cdot 6, 2 \cdot 4 \cdot 6 \cdot 8, 2 \cdot 4 \cdot 6 \cdot 8 \cdot 10, \dots\} = \{2, 8, 48, 384, 3840, \dots\}$.

19. $a_n = \frac{3 + 5n^2}{n + n^2} = \frac{(3 + 5n^2)/n^2}{(n + n^2)/n^2} = \frac{5 + 3/n^2}{1 + 1/n}$, so $a_n \rightarrow \frac{5 + 0}{1 + 0} = 5$ as $n \rightarrow \infty$. Converges

21. Because the natural exponential function is continuous at 0, Theorem 7 enables us to write

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} e^{1/n} = e^{\lim_{n \rightarrow \infty} (1/n)} = e^0 = 1. \text{ Converges}$$

27. $a_n = \cos(n/2)$. This sequence diverges since the terms don't approach any particular real number as $n \rightarrow \infty$.

The terms take on values between -1 and 1 .

29. $a_n = \frac{(2n-1)!}{(2n+1)!} = \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{1}{(2n+1)(2n)} \rightarrow 0$ as $n \rightarrow \infty$. Converges

45. $a_n = \frac{n!}{2^n} = \frac{1}{2} \cdot \frac{2}{2} \cdot \frac{3}{2} \cdot \dots \cdot \frac{(n-1)}{2} \cdot \frac{n}{2} \geq \frac{1}{2} \cdot \frac{n}{2}$ [for $n > 1$] $= \frac{n}{4} \rightarrow \infty$ as $n \rightarrow \infty$, so $\{a_n\}$ diverges.