

3. The distance from a point to the  $xz$ -plane is the absolute value of the  $y$ -coordinate of the point.  $Q(-5, -1, 4)$  has the  $y$ -coordinate with the smallest absolute value, so  $Q$  is the point closest to the  $xz$ -plane.  $R(0, 3, 8)$  must lie in the  $yz$ -plane since the distance from  $R$  to the  $yz$ -plane, given by the  $x$ -coordinate of  $R$ , is 0.

10. (a) The distance from a point to the  $xy$ -plane is the absolute value of the  $z$ -coordinate of the point. Thus, the distance is  $|-5| = 5$ .

(b) Similarly, the distance to the  $yz$ -plane is the absolute value of the  $x$ -coordinate of the point:  $|3| = 3$ .

(c) The distance to the  $xz$ -plane is the absolute value of the  $y$ -coordinate of the point:  $|7| = 7$ .

(d) The point on the  $x$ -axis closest to  $(3, 7, -5)$  is the point  $(3, 0, 0)$ . (Approach the  $x$ -axis perpendicularly.)

The distance from  $(3, 7, -5)$  to the  $x$ -axis is the distance between these two points:

$$\sqrt{(3-3)^2 + (7-0)^2 + (-5-0)^2} = \sqrt{74} \approx 8.60.$$

(e) The point on the  $y$ -axis closest to  $(3, 7, -5)$  is  $(0, 7, 0)$ . The distance between these points is

$$\sqrt{(3-0)^2 + (7-7)^2 + (-5-0)^2} = \sqrt{34} \approx 5.83.$$

(f) The point on the  $z$ -axis closest to  $(3, 7, -5)$  is  $(0, 0, -5)$ . The distance between these points is

$$\sqrt{(3-0)^2 + (7-0)^2 + [-5-(-5)]^2} = \sqrt{58} \approx 7.62.$$

11. An equation of the sphere with center  $(1, -4, 3)$  and radius 5 is  $(x-1)^2 + [y-(-4)]^2 + (z-3)^2 = 5^2$  or

$(x-1)^2 + (y+4)^2 + (z-3)^2 = 25$ . The intersection of this sphere with the  $xz$ -plane is the set of points on the sphere

whose  $y$ -coordinate is 0. Putting  $y = 0$  into the equation, we have  $(x-1)^2 + 4^2 + (z-3)^2 = 25$ ,  $y = 0$  or

$(x-1)^2 + (z-3)^2 = 9$ ,  $y = 0$ , which represents a circle in the  $xz$ -plane with center  $(1, 0, 3)$  and radius 3.

14. If the sphere passes through the origin, the radius of the sphere must be the distance from the origin to the point  $(1, 2, 3)$ :

$r = \sqrt{(1-0)^2 + (2-0)^2 + (3-0)^2} = \sqrt{14}$ . Then an equation of the sphere is  $(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$ .

17. Completing squares in the equation  $2x^2 - 8x + 2y^2 + 2z^2 + 24z = 1$  gives

$$2(x^2 - 4x + 4) + 2y^2 + 2(z^2 + 12z + 36) = 1 + 8 + 72 \Rightarrow 2(x-2)^2 + 2y^2 + 2(z+6)^2 = 81 \Rightarrow$$

$(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2}$ , which we recognize as an equation of a sphere with center  $(2, 0, -6)$  and radius

$$\sqrt{\frac{81}{2}} = 9/\sqrt{2}.$$

21. (a) Since the sphere touches the  $xy$ -plane, its radius is the distance from its center,  $(2, -3, 6)$ , to the  $xy$ -plane, namely 6.

Therefore  $r = 6$  and an equation of the sphere is  $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 6^2 = 36$ .

(b) The radius of this sphere is the distance from its center  $(2, -3, 6)$  to the  $yz$ -plane, which is 2. Therefore, an equation is

$$(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 4.$$

(c) Here the radius is the distance from the center  $(2, -3, 6)$  to the  $xz$ -plane, which is 3. Therefore, an equation is

$$(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 9.$$

28. The equation  $z^2 = 1 \Leftrightarrow z = \pm 1$  represents two horizontal planes;  $z = 1$  is parallel to the  $xy$ -plane, one unit above it, and  $z = -1$  is one unit below it.

31. Here  $x^2 + z^2 \leq 9$  or equivalently  $\sqrt{x^2 + z^2} \leq 3$  which describes the set of all points in  $\mathbb{R}^3$  whose distance from the  $y$ -axis is at most 3. Thus, the inequality represents the region consisting of all points on or inside a circular cylinder of radius 3 with axis the  $y$ -axis.

36. The solid sphere itself is represented by  $\sqrt{x^2 + y^2 + z^2} \leq 2$ . Since we want only the upper hemisphere, we restrict the  $z$ -coordinate to nonnegative values. Then inequalities describing the region are  $\sqrt{x^2 + y^2 + z^2} \leq 2, z \geq 0$ , or  $x^2 + y^2 + z^2 \leq 4, z \geq 0$ .