6. $\mathbf{a}\cdot\mathbf{b} = \langle s, 2s, 3s\rangle \cdot \langle t, -t, 5t\rangle = (s)(t) + (2s)(-t) + (3s)(5t) = st - 2st + 15st = 14st$

7. $\mathbf{a}\cdot\mathbf{b} = (\mathbf{i} - 2\,\mathbf{j} + 3\,\mathbf{k})\cdot(5\,\mathbf{i} + 9\,\mathbf{k}) = (1)(5) + (-2)(0) + (3)(9) = 32$

11. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all unit vectors, so the triangle is an equilateral triangle. Thus the angle between $\mathbf{u}$ and $\mathbf{v}$ is $60°$ and $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\cos 60° = (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$. If $\mathbf{w}$ is moved so it has the same initial point as $\mathbf{u}$, we can see that the angle between them is $120°$ and we have $\mathbf{u}\cdot\mathbf{w} = |\mathbf{u}|\,|\mathbf{w}|\cos 120° = (1)(1)\left(-\frac{1}{2}\right) = -\frac{1}{2}$.

17. $|\mathbf{a}| = \sqrt{3^2 + (-1)^2 + 5^2} = \sqrt{35}$, $|\mathbf{b}| = \sqrt{(-2)^2 + 4^2 + 3^2} = \sqrt{29}$, and $\mathbf{a}\cdot\mathbf{b} = (3)(-2) + (-1)(4) + (5)(3) = 5$. Then $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{5}{\sqrt{35}\cdot\sqrt{29}} = \dfrac{5}{\sqrt{1015}}$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = \cos^{-1}\left(\frac{5}{\sqrt{1015}}\right) \approx 81°$.

19. $|\mathbf{a}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$, $|\mathbf{b}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$, and $\mathbf{a}\cdot\mathbf{b} = (0)(1) + (1)(2) + (1)(-3) = -1$. Then $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{-1}{\sqrt{2}\cdot\sqrt{14}} = \dfrac{-1}{2\sqrt{7}}$ and $\theta = \cos^{-1}\left(-\dfrac{1}{2\sqrt{7}}\right) \approx 101°$.

23. (a) $\mathbf{a}\cdot\mathbf{b} = (-5)(6) + (3)(-8) + (7)(2) = -40 \neq 0$, so $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal. Also, since $\mathbf{a}$ is not a scalar multiple of $\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

(b) $\mathbf{a}\cdot\mathbf{b} = (4)(-3) + (6)(2) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

(c) $\mathbf{a}\cdot\mathbf{b} = (-1)(3) + (2)(4) + (5)(-1) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

(d) Because $\mathbf{a} = -\frac{2}{3}\,\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are parallel.

26. $\langle -6, b, 2\rangle$ and $\langle b, b^2, b\rangle$ are orthogonal when $\langle -6, b, 2\rangle \cdot \langle b, b^2, b\rangle = 0 \quad\Leftrightarrow\quad (-6)(b) + (b)(b^2) + (2)(b) = 0 \quad\Leftrightarrow$ $b^3 - 4b = 0 \quad\Leftrightarrow\quad b(b+2)(b-2) = 0 \quad\Leftrightarrow\quad b = 0$ or $b = \pm 2$.

36. $|\mathbf{a}| = \sqrt{1^2 + 2^2} = \sqrt{5}$, so the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{1(-4) + 2\cdot 1}{\sqrt{5}} = -\dfrac{2}{\sqrt{5}}$ and the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = -\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\langle 1, 2\rangle = \left\langle -\frac{2}{5}, -\frac{4}{5}\right\rangle$.

37. $|\mathbf{a}| = \sqrt{9 + 36 + 4} = 7$ so the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \frac{1}{7}(3 + 12 - 6) = \frac{9}{7}$. The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{9}{7}\dfrac{\mathbf{a}}{|\mathbf{a}|} = \frac{9}{7}\cdot\frac{1}{7}\langle 3, 6, -2\rangle = \frac{9}{49}\langle 3, 6, -2\rangle = \left\langle \frac{27}{49}, \frac{54}{49}, -\frac{18}{49}\right\rangle$.

44. (a) $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \text{comp}_{\mathbf{b}}\,\mathbf{a} \quad \Leftrightarrow \quad \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|} \quad \Leftrightarrow \quad \dfrac{1}{|\mathbf{a}|} = \dfrac{1}{|\mathbf{b}|}$ or $\mathbf{a}\cdot\mathbf{b} = 0 \quad \Leftrightarrow \quad |\mathbf{b}| = |\mathbf{a}|$ or $\mathbf{a}\cdot\mathbf{b} = 0$.

That is, if $\mathbf{a}$ and $\mathbf{b}$ are orthogonal or if they have the same length.

(b) $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \text{proj}_{\mathbf{b}}\,\mathbf{a} \quad \Leftrightarrow \quad \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} = \dfrac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|^2}\mathbf{b} \quad \Leftrightarrow \quad \mathbf{a}\cdot\mathbf{b} = 0$ or $\dfrac{\mathbf{a}}{|\mathbf{a}|^2} = \dfrac{\mathbf{b}}{|\mathbf{b}|^2}$.

But $\dfrac{\mathbf{a}}{|\mathbf{a}|^2} = \dfrac{\mathbf{b}}{|\mathbf{b}|^2} \quad \Rightarrow \quad \dfrac{|\mathbf{a}|}{|\mathbf{a}|^2} = \dfrac{|\mathbf{b}|}{|\mathbf{b}|^2} \quad \Rightarrow \quad |\mathbf{a}| = |\mathbf{b}|$. Substituting this into the previous equation gives $\mathbf{a} = \mathbf{b}$.

So $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \text{proj}_{\mathbf{b}}\,\mathbf{a} \quad \Leftrightarrow \quad \mathbf{a}$ and $\mathbf{b}$ are orthogonal, or they are equal.

47. Here $|\mathbf{D}| = 80$ ft, $|\mathbf{F}| = 30$ lb, and $\theta = 40°$. Thus

$W = \mathbf{F}\cdot\mathbf{D} = |\mathbf{F}|\,|\mathbf{D}|\cos\theta = (30)(80)\cos 40° = 2400\cos 40° \approx 1839$ ft-lb.