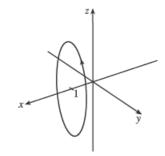
## Solutions 13.1--Part 1--Sping 2008

2. The component functions  $\frac{t-2}{t+2}$ ,  $\sin t$ , and  $\ln(9-t^2)$  are all defined when  $t \neq -2$  and  $9-t^2 > 0 \implies -3 < t < 3$ , so the domain of  ${\bf r}$  is  $(-3,-2) \cup (-2,3)$ .

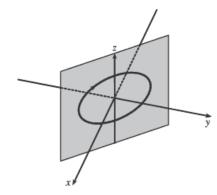
$$5. \lim_{t \to 0} e^{-3t} = e^0 = 1, \lim_{t \to 0} \frac{t^2}{\sin^2 t} = \lim_{t \to 0} \frac{1}{\frac{\sin^2 t}{t^2}} = \frac{1}{\lim_{t \to 0} \frac{\sin^2 t}{t^2}} = \frac{1}{\left(\lim_{t \to 0} \frac{\sin t}{t}\right)^2} = \frac{1}{1^2} = 1$$

and  $\lim_{t\to 0} \cos 2t = \cos 0 = 1$ . Thus the given limit equals  $\mathbf{i} + \mathbf{j} + \mathbf{k}$ .

11. The corresponding parametric equations are x=1,  $y=\cos t$ ,  $z=2\sin t$ . Eliminating the parameter in y and z gives  $y^2+(z/2)^2=\cos^2 t+\sin^2 t=1$  or  $y^2+z^2/4=1$ . Since x=1, the curve is an ellipse centered at (1,0,0) in the plane x=1.



14. If  $x = \cos t$ ,  $y = -\cos t$ ,  $z = \sin t$ , then  $x^2 + z^2 = 1$  and  $y^2 + z^2 = 1$ , so the curve is contained in the intersection of circular cylinders along the x- and y-axes. Furthermore, y = -x, so the curve is an ellipse in the plane y = -x, centered at the origin.



- 20.  $x=t,\ y=t^2,\ z=e^{-t}$ . At any point on the curve,  $y=x^2$ . So the curve lies on the parabolic cylinder  $y=x^2$ . Note that y and z are positive for all t, and the point (0,0,1) is on the curve (when t=0). As  $t\to\infty$ ,  $(x,y,z)\to(\infty,\infty,0)$ , while as  $t\to-\infty$ ,  $(x,y,z)\to(-\infty,\infty,\infty)$ , so the graph must be  $\Pi$ .
- 24.  $x = \cos t$ ,  $y = \sin t$ ,  $z = \ln t$ .  $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$ , so the curve lies on a circular cylinder with axis the z-axis. As  $t \to 0$ ,  $z \to -\infty$ , so the graph is III.