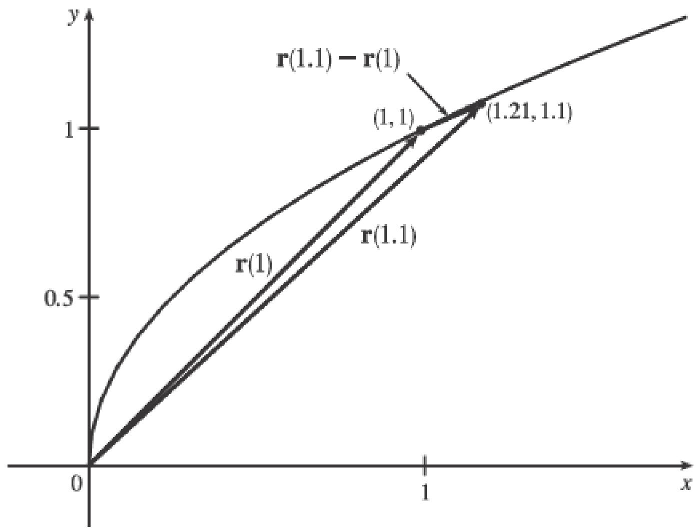
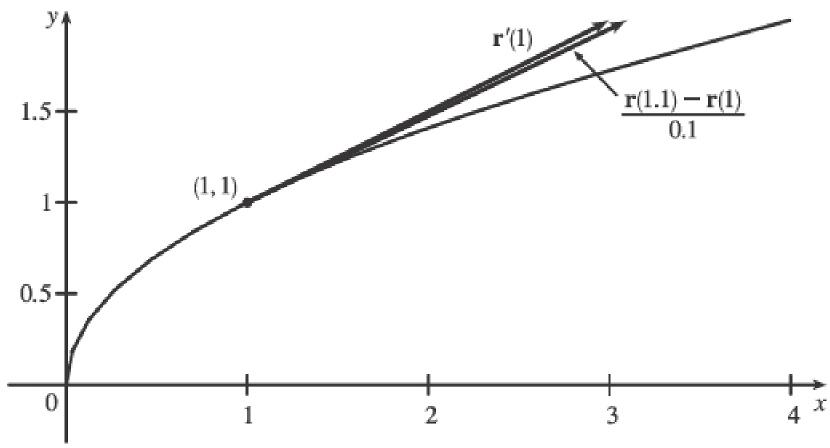
2. (a) The curve can be represented by the parametric equations $x = t^2$, $y = t$, $0 \le t \le 2$. Eliminating the parameter, we have $x = y^2$, $0 \le y \le 2$, a portion of which we graph here, along with the vectors $\mathbf{r}(1)$, $\mathbf{r}(1.1)$, and $\mathbf{r}(1.1) - \mathbf{r}(1)$.



(b) Since $\mathbf{r}(t) = \langle t^2, t \rangle$, we differentiate components, giving $\mathbf{r}'(t) = \langle 2t, 1 \rangle$, so $\mathbf{r}'(1) = \langle 2, 1 \rangle$.

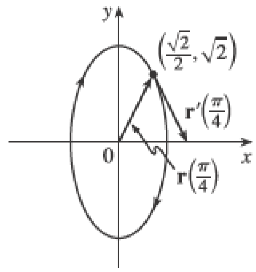
$$\frac{\mathbf{r}(1.1) - \mathbf{r}(1)}{0.1} = \frac{\langle 1.21, 1.1 \rangle - \langle 1, 1 \rangle}{0.1} = 10 \langle 0.21, 0.1 \rangle = \langle 2.1, 1 \rangle.$$



As we can see from the graph, these vectors are very close in length and direction. $\mathbf{r}'(1)$ is defined to be $\lim_{h \to 0} \frac{\mathbf{r}(1+h) - \mathbf{r}(1)}{h}$, and we recognize $\frac{\mathbf{r}(1.1) - \mathbf{r}(1)}{0.1}$ as the expression after the limit sign with $h = 0.1$. Since h is close to 0, we would expect $\frac{\mathbf{r}(1.1) - \mathbf{r}(1)}{0.1}$ to be a vector close to $\mathbf{r}'(1)$.

5. $x = \sin t$, $y = 2\cos t$ so $x^2 + (y/2)^2 = 1$ and the curve is an ellipse.

(a), (c)

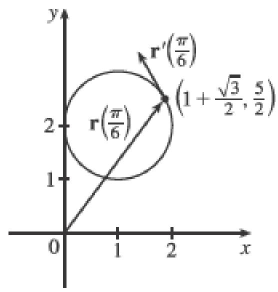


(b) $\mathbf{r}'(t) = \cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$,

$$\mathbf{r}'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$$

8. $x = 1 + \cos t$, $y = 2 + \sin t$ so $(x-1)^2 + (y-2)^2 = 1$ and the curve is a circle.

(a), (c)



(b) $\mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j}$,

$$\mathbf{r}'\left(\frac{\pi}{6}\right) = -\frac{1}{2}\,\mathbf{i} + \frac{\sqrt{3}}{2}\,\mathbf{j}$$

15. $\mathbf{r}'(t) = \mathbf{0} + \mathbf{b} + 2t\,\mathbf{c} = \mathbf{b} + 2t\,\mathbf{c}$ by Formulas 1 and 3 of Theorem 3.

16. To find $\mathbf{r}'(t)$, we first expand $\mathbf{r}(t) = t\,\mathbf{a} \times (\mathbf{b} + t\,\mathbf{c}) = t(\mathbf{a} \times \mathbf{b}) + t^2(\mathbf{a} \times \mathbf{c})$, so $\mathbf{r}'(t) = \mathbf{a} \times \mathbf{b} + 2t(\mathbf{a} \times \mathbf{c})$.

17. $\mathbf{r}'(t) = \langle -te^{-t} + e^{-t}, 2/(1+t^2), 2e^t \rangle \quad \Rightarrow \quad \mathbf{r}'(0) = \langle 1, 2, 2 \rangle$. So $|\mathbf{r}'(0)| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$ and

$$\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \tfrac{1}{3}\langle 1, 2, 2 \rangle = \left\langle \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3} \right\rangle.$$

23. The vector equation for the curve is $\mathbf{r}(t) = \langle 1 + 2\sqrt{t}, t^3 - t, t^3 + t \rangle$, so $\mathbf{r}'(t) = \langle 1/\sqrt{t}, 3t^2 - 1, 3t^2 + 1 \rangle$. The point $(3, 0, 2)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle 1, 2, 4 \rangle$. Thus, the tangent line goes through the point $(3, 0, 2)$ and is parallel to the vector $\langle 1, 2, 4 \rangle$. Parametric equations are $x = 3 + t$, $y = 2t$, $z = 2 + 4t$.

33.
$$\int_0^1 (16t^3\,\mathbf{i} - 9t^2\,\mathbf{j} + 25t^4\,\mathbf{k})\,dt = \left(\int_0^1 16t^3\,dt\right)\mathbf{i} - \left(\int_0^1 9t^2\,dt\right)\mathbf{j} + \left(\int_0^1 25t^4\,dt\right)\mathbf{k}$$
$$= \left[4t^4\right]_0^1\,\mathbf{i} - \left[3t^3\right]_0^1\,\mathbf{j} + \left[5t^5\right]_0^1\,\mathbf{k} = 4\,\mathbf{i} - 3\,\mathbf{j} + 5\,\mathbf{k}$$

35.
$$\int_0^{\pi/2} (3\sin^2 t\,\cos t\,\mathbf{i} + 3\sin t\,\cos^2 t\,\mathbf{j} + 2\sin t\,\cos t\,\mathbf{k})\,dt$$
$$= \left(\int_0^{\pi/2} 3\sin^2 t\,\cos t\,dt\right)\mathbf{i} + \left(\int_0^{\pi/2} 3\sin t\,\cos^2 t\,dt\right)\mathbf{j} + \left(\int_0^{\pi/2} 2\sin t\,\cos t\,dt\right)\mathbf{k}$$
$$= \left[\sin^3 t\right]_0^{\pi/2}\,\mathbf{i} + \left[-\cos^3 t\right]_0^{\pi/2}\,\mathbf{j} + \left[\sin^2 t\right]_0^{\pi/2}\,\mathbf{k} = (1-0)\,\mathbf{i} + (0+1)\,\mathbf{j} + (1-0)\,\mathbf{k} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$