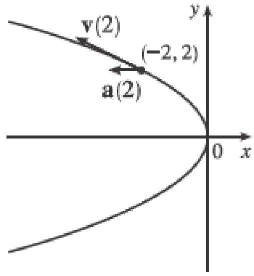
**3.** $\mathbf{r}(t) = \langle -\frac{1}{2}t^2, t \rangle \quad \Rightarrow$ At $t = 2$:

$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -t, 1 \rangle$ $\quad \mathbf{v}(2) = \langle -2, 1 \rangle$

$\mathbf{a}(t) = \mathbf{r}''(t) = \langle -1, 0 \rangle$ $\quad \mathbf{a}(2) = \langle -1, 0 \rangle$

$|\mathbf{v}(t)| = \sqrt{t^2 + 1}$



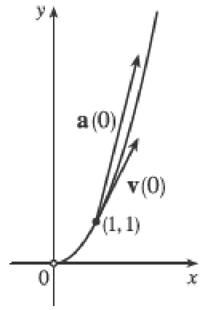

**6.** $\mathbf{r}(t) = e^t\,\mathbf{i} + e^{2t}\,\mathbf{j} \quad \Rightarrow$ At $t = 0$:

$\mathbf{v}(t) = e^t\,\mathbf{i} + 2e^{2t}\,\mathbf{j}$ $\quad \mathbf{v}(0) = \mathbf{i} + 2\,\mathbf{j}$

$\mathbf{a}(t) = e^t\,\mathbf{i} + 4e^{2t}\,\mathbf{j}$ $\quad \mathbf{a}(0) = \mathbf{i} + 4\,\mathbf{j}$

$|\mathbf{v}(t)| = \sqrt{e^{2t} + 4e^{4t}} = e^t\sqrt{1 + 4e^{2t}}$

Notice that $y = e^{2t} = \left(e^t\right)^2 = x^2$, so the particle travels along a parabola, but $x = e^t$, so $x > 0$.




**11.** $\mathbf{r}(t) = \sqrt{2}\,t\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k} \quad \Rightarrow \quad \mathbf{v}(t) = \mathbf{r}'(t) = \sqrt{2}\,\mathbf{i} + e^t\,\mathbf{j} - e^{-t}\,\mathbf{k}, \quad \mathbf{a}(t) = \mathbf{v}'(t) = e^t\,\mathbf{j} + e^{-t}\,\mathbf{k},$

$|\mathbf{v}(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$.

**15.** $\mathbf{a}(t) = \mathbf{i} + 2\,\mathbf{j} \quad \Rightarrow \quad \mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \int (\mathbf{i} + 2\,\mathbf{j})\,dt = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{C}$ and $\mathbf{k} = \mathbf{v}\,(0) = \mathbf{C}$,

so $\mathbf{C} = \mathbf{k}$ and $\mathbf{v}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}$. $\quad \mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int (t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k})\,dt = \frac{1}{2}t^2\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k} + \mathbf{D}$.

But $\mathbf{i} = \mathbf{r}\,(0) = \mathbf{D}$, so $\mathbf{D} = \mathbf{i}$ and $\mathbf{r}(t) = \left(\frac{1}{2}t^2 + 1\right)\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}$.

**25.** As in Example 5, $\mathbf{r}(t) = (v_0 \cos 45°)t\,\mathbf{i} + \left[(v_0 \sin 45°)t - \frac{1}{2}gt^2\right]\mathbf{j} = \frac{1}{2}\left[v_0\sqrt{2}\,t\,\mathbf{i} + \left(v_0\sqrt{2}\,t - gt^2\right)\mathbf{j}\right]$. Then the ball lands at $t = \dfrac{v_0\sqrt{2}}{g}$ s. Now since it lands 90 m away, $90 = \frac{1}{2}v_0\,\sqrt{2}\,\dfrac{v_0\,\sqrt{2}}{g}$ or $v_0^2 = 90g$ and the initial velocity is $v_0 = \sqrt{90g} \approx 30$ m/s.

**27.** Let $\alpha$ be the angle of elevation. Then $v_0 = 150$ m/s and from Example 5, the horizontal distance traveled by the projectile is $d = \dfrac{v_0^2 \sin 2\alpha}{g}$. Thus $\dfrac{150^2 \sin 2\alpha}{g} = 800 \quad \Rightarrow \quad \sin 2\alpha = \dfrac{800g}{150^2} \approx 0.3484 \quad \Rightarrow \quad 2\alpha \approx 20.4°$ or $180 - 20.4 = 159.6°$.

Two angles of elevation then are $\alpha \approx 10.2°$ and $\alpha \approx 79.8°$.

**28.** Here $v_0 = 115$ ft/s, the angle of elevation is $\alpha = 50°$, and if we place the origin at home plate, then $\mathbf{r}(0) = 3\,\mathbf{j}$. As in Example 5, we have $\mathbf{r}(t) = -\frac{1}{2}gt^2\,\mathbf{j} + t\,\mathbf{v}_0 + \mathbf{D}$ where $\mathbf{D} = \mathbf{r}(0) = 3\,\mathbf{j}$ and $\mathbf{v}_0 = v_0 \cos\alpha\,\mathbf{i} + v_0 \sin\alpha\,\mathbf{j}$, so $\mathbf{r}(t) = (v_0 \cos\alpha)t\,\mathbf{i} + \left[(v_0 \sin\alpha)t - \frac{1}{2}gt^2 + 3\right]\mathbf{j}$. Thus, parametric equations for the trajectory of the ball are $x = (v_0 \cos\alpha)t$, $y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 + 3$. The ball reaches the fence when $x = 400 \Rightarrow$ $(v_0 \cos\alpha)t = 400 \Rightarrow t = \dfrac{400}{v_0 \cos\alpha} = \dfrac{400}{115\cos 50°} \approx 5.41$ s. At this time, the height of the ball is $y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 + 3 \approx (115 \sin 50°)(5.41) - \frac{1}{2}(32)(5.41)^2 + 3 \approx 11.2$ ft. Since the fence is 10 ft high, the ball clears the fence.

**33.** $\mathbf{r}(t) = (3t - t^3)\,\mathbf{i} + 3t^2\,\mathbf{j} \quad\Rightarrow\quad \mathbf{r}'(t) = (3 - 3t^2)\,\mathbf{i} + 6t\,\mathbf{j}$,

$|\mathbf{r}'(t)| = \sqrt{(3-3t^2)^2 + (6t)^2} = \sqrt{9 + 18t^2 + 9t^4} = \sqrt{(3-3t^2)^2} = 3 + 3t^2$,

$\mathbf{r}''(t) = -6t\,\mathbf{i} + 6\,\mathbf{j}$, $\mathbf{r}'(t) \times \mathbf{r}''(t) = (18 + 18t^2)\,\mathbf{k}$. Then Equation 9 gives

$a_T = \dfrac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \dfrac{(3-3t^2)(-6t) + (6t)(6)}{3 + 3t^2} = \dfrac{18t + 18t^3}{3 + 3t^2} = \dfrac{18t(1+t^2)}{3(1+t^2)} = 6t$ $\left[\text{or by Equation 8, } a_T = v' = \dfrac{d}{dt}\left[3 + 3t^2\right] = 6t\right]$ and Equation 10 gives $a_N = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \dfrac{18 + 18t^2}{3 + 3t^2} = \dfrac{18(1+t^2)}{3(1+t^2)} = 6$.

**34.** $\mathbf{r}(t) = (1 + t)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j} \quad\Rightarrow\quad \mathbf{r}'(t) = \mathbf{i} + (2t - 2)\,\mathbf{j}$, $\quad |\mathbf{r}'(t)| = \sqrt{1^2 + (2t-2)^2} = \sqrt{4t^2 - 8t + 5}$,

$\mathbf{r}''(t) = 2\,\mathbf{j}$, $\quad \mathbf{r}'(t) \times \mathbf{r}''(t) = 2\,\mathbf{k}$. Then Equation 9 gives $a_T = \dfrac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \dfrac{2(2t-2)}{\sqrt{4t^2 - 8t + 5}}$ and Equation 10 gives $a_N = \dfrac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \dfrac{2}{\sqrt{4t^2 - 8t + 5}}$.