3. $z = f(x,y) = \sqrt{xy} \quad \Rightarrow \quad f_x(x,y) = \frac{1}{2}(xy)^{-1/2} \cdot y = \frac{1}{2}\sqrt{y/x}$, $f_y(x,y) = \frac{1}{2}(xy)^{-1/2} \cdot x = \frac{1}{2}\sqrt{x/y}$, so $f_x(1,1) = \frac{1}{2}$ and $f_y(1,1) = \frac{1}{2}$. Thus an equation of the tangent plane is $z - 1 = f_x(1,1)(x-1) + f_y(1,1)(y-1) \quad \Rightarrow$ $z - 1 = \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$ or $x + y - 2z = 0$.

4. $z = f(x,y) = y \ln x \quad \Rightarrow \quad f_x(x,y) = y/x$, $f_y(x,y) = \ln x$, so $f_x(1,4) = 4$, $f_y(1,4) = 0$, and an equation of the tangent plane is $z - 0 = f_x(1,4)(x-1) + f_y(1,4)(y-4) \quad \Rightarrow \quad z = 4(x-1) + 0(y-4)$ or $z = 4x - 4$.

12. $f(x,y) = x^3 y^4$. The partial derivatives are $f_x(x,y) = 3x^2y^4$ and $f_y(x,y) = 4x^3y^3$, so $f_x(1,1) = 3$ and $f_y(1,1) = 4$. Both f_x and f_y are continuous functions, so f is differentiable at $(1,1)$ by Theorem 8. The linearization of f at $(1,1)$ is given by $L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1) = 1 + 3(x-1) + 4(y-1) = 3x + 4y - 6$.

13. $f(x,y) = \dfrac{x}{x+y}$. The partial derivatives are $f_x(x,y) = \dfrac{1(x+y) - x(1)}{(x+y)^2} = y/(x+y)^2$ and $f_y(x,y) = x(-1)(x+y)^{-2} \cdot 1 = -x/(x+y)^2$, so $f_x(2,1) = \frac{1}{9}$ and $f_y(2,1) = -\frac{2}{9}$. Both f_x and f_y are continuous functions for $y \neq -x$, so f is differentiable at $(2,1)$ by Theorem 8. The linearization of f at $(2,1)$ is given by $L(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1) = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1) = \frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$.

17. Let $f(x,y) = \dfrac{2x+3}{4y+1}$. Then $f_x(x,y) = \dfrac{2}{4y+1}$ and $f_y(x,y) = (2x+3)(-1)(4y+1)^{-2}(4) = \dfrac{-8x-12}{(4y+1)^2}$. Both f_x and f_y are continuous functions for $y \neq -\frac{1}{4}$, so by Theorem 8, f is differentiable at $(0,0)$. We have $f_x(0,0) = 2$, $f_y(0,0) = -12$ and the linear approximation of f at $(0,0)$ is $f(x,y) \approx f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) = 3 + 2x - 12y$.

19. $f(x,y) = \sqrt{20 - x^2 - 7y^2} \quad \Rightarrow \quad f_x(x,y) = -\dfrac{x}{\sqrt{20 - x^2 - 7y^2}}$ and $f_y(x,y) = -\dfrac{7y}{\sqrt{20 - x^2 - 7y^2}}$, so $f_x(2,1) = -\frac{2}{3}$ and $f_y(2,1) = -\frac{7}{3}$. Then the linear approximation of f at $(2,1)$ is given by $f(x,y) \approx f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1) = 3 - \frac{2}{3}(x-2) - \frac{7}{3}(y-1) = -\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$.
Thus $f(1.95, 1.08) \approx -\frac{2}{3}(1.95) - \frac{7}{3}(1.08) + \frac{20}{3} = 2.84\overline{6}$.

22. From the table, $f(40,20) = 28$. To estimate $f_v(40,20)$ and $f_t(40,20)$ we follow the procedure used in Exercise 15.3.4 [ET 14.3.4]. Since $f_v(40,20) = \lim_{h\to 0} \dfrac{f(40+h,20) - f(40,20)}{h}$, we approximate this quantity with $h = \pm 10$ and use the values given in the table:

$$f_v(40,20) \approx \frac{f(50,20) - f(40,20)}{10} = \frac{40-28}{10} = 1.2, \qquad f_v(40,20) \approx \frac{f(30,20) - f(40,20)}{-10} = \frac{17-28}{-10} = 1.1$$

Averaging these values gives $f_v(40,20) \approx 1.15$. Similarly, $f_t(40,20) = \lim_{h\to 0} \dfrac{f(40,20+h) - f(40,20)}{h}$, so we use $h = 10$ and $h = -5$:

$$f_t(40,20) \approx \frac{f(40,30) - f(40,20)}{10} = \frac{31-28}{10} = 0.3, \qquad f_t(40,20) \approx \frac{f(40,15) - f(40,20)}{-5} = \frac{25-28}{-5} = 0.6$$

Averaging these values gives $f_t(40,15) \approx 0.45$. The linear approximation, then, is

$$f(v,t) \approx f(40,20) + f_v(40,20)(v-40) + f_t(40,20)(t-20) \approx 28 + 1.15(v-40) + 0.45(t-20)$$

When $v = 43$ and $t = 24$, we estimate $f(43,24) \approx 28 + 1.15(43-40) + 0.45(24-20) = 33.25$, so we would expect the wave heights to be approximately 33.25 ft.

25. $z = x^3 \ln(y^2) \quad \Rightarrow \quad dz = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy = 3x^2 \ln(y^2)\, dx + x^3 \cdot \dfrac{1}{y^2}(2y)\, dy = 3x^2 \ln(y^2)\, dx + \dfrac{2x^3}{y}dy$

26. $v = y \cos xy \quad \Rightarrow$

$dv = \dfrac{\partial v}{\partial x}dx + \dfrac{\partial v}{\partial y}dy = y(-\sin xy)y\, dx + [y(-\sin xy)x + \cos xy]\, dy = -y^2 \sin xy\, dx + (\cos xy - xy \sin xy)\, dy$

33. $dA = \dfrac{\partial A}{\partial x}\, dx + \dfrac{\partial A}{\partial y}\, dy = y\, dx + x\, dy$ and $|\Delta x| \le 0.1$, $|\Delta y| \le 0.1$. We use $dx = 0.1$, $dy = 0.1$ with $x = 30$, $y = 24$; then the maximum error in the area is about $dA = 24(0.1) + 30(0.1) = 5.4\ \text{cm}^2$.

42. $\mathbf{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2 \rangle \quad \Rightarrow \quad \mathbf{r}_1'(t) = \langle 3, -2t, -4+2t \rangle$, $\mathbf{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1 \rangle \quad \Rightarrow$ $\mathbf{r}_2'(u) = \langle 2u, 6u^2, 2 \rangle$. Both curves pass through P since $\mathbf{r}_1(0) = \mathbf{r}_2(1) = \langle 2,1,3 \rangle$, so the tangent vectors $\mathbf{r}_1'(0) = \langle 3, 0, -4 \rangle$ and $\mathbf{r}_2'(1) = \langle 2, 6, 2 \rangle$ are both parallel to the tangent plane to S at P. A normal vector for the tangent plane is $\mathbf{r}_1'(0) \times \mathbf{r}_2'(1) = \langle 3, 0, -4 \rangle \times \langle 2, 6, 2 \rangle = \langle 24, -14, 18 \rangle$, so an equation of the tangent plane is $24(x-2) - 14(y-1) + 18(z-3) = 0$ or $12x - 7y + 9z = 44$.