3. $\int_1^3 \int_0^1 (1+4xy)\,dx\,dy = \int_1^3 \left[x + 2x^2y\right]_{x=0}^{x=1} dy = \int_1^3 (1+2y)\,dy = \left[y+y^2\right]_1^3 = (3+9)-(1+1) = 10$

5. $\int_0^2 \int_0^{\pi/2} x \sin y\,dy\,dx = \int_0^2 x\,dx \int_0^{\pi/2} \sin y\,dy$ [as in Example 5] $= \left[\frac{x^2}{2}\right]_0^2 \left[-\cos y\right]_0^{\pi/2} = (2-0)(0+1) = 2$

8. $\displaystyle\int_0^1 \int_1^2 \frac{xe^x}{y}\,dy\,dx = \int_0^1 xe^x\,dx \int_1^2 \frac{1}{y}\,dy$ [as in Example 5] $= \left[xe^x - e^x\right]_0^1 \left[\ln|y|\right]_1^2$ [by integrating by parts]

$= [(e-e)-(0-1)](\ln 2 - 0) = \ln 2$

9. $\displaystyle\int_1^4 \int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right) dy\,dx = \int_1^4 \left[x \ln|y| + \frac{1}{x}\cdot\frac{1}{2}y^2\right]_{y=1}^{y=2} dx = \int_1^4 \left(x\ln 2 + \frac{3}{2x}\right) dx = \left[\tfrac{1}{2}x^2 \ln 2 + \tfrac{3}{2}\ln|x|\right]_1^4$

$= 8\ln 2 + \frac{3}{2}\ln 4 - \frac{1}{2}\ln 2 = \frac{15}{2}\ln 2 + 3\ln 4^{1/2} = \frac{21}{2}\ln 2$

16. $\iint_R \cos(x+2y)\,dA = \int_0^\pi \int_0^{\pi/2} \cos(x+2y)\,dy\,dx = \int_0^\pi \left[\frac{1}{2}\sin(x+2y)\right]_{y=0}^{y=\pi/2} dx = \frac{1}{2}\int_0^\pi (\sin(x+\pi) - \sin x)\,dx$

$= \frac{1}{2}\left[-\cos(x+\pi) + \cos x\right]_0^\pi = \frac{1}{2}\left[-\cos 2\pi + \cos\pi - (-\cos\pi + \cos 0)\right]$

$= \frac{1}{2}(-1-1-(1+1)) = -2$

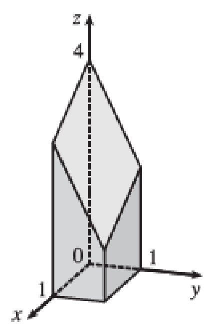
19. $\int_0^{\pi/6} \int_0^{\pi/3} x \sin(x+y)\,dy\,dx$

$= \int_0^{\pi/6} \left[-x\cos(x+y)\right]_{y=0}^{y=\pi/3} dx = \int_0^{\pi/6} \left[x\cos x - x\cos\left(x+\frac{\pi}{3}\right)\right] dx$

$= x\left[\sin x - \sin\left(x+\frac{\pi}{3}\right)\right]_0^{\pi/6} - \int_0^{\pi/6} \left[\sin x - \sin\left(x+\frac{\pi}{3}\right)\right] dx$ [by integrating by parts separately for each term]

$= \frac{\pi}{6}\left[\frac{1}{2}-1\right] - \left[-\cos x + \cos\left(x+\frac{\pi}{3}\right)\right]_0^{\pi/6} = -\frac{\pi}{12} - \left[-\frac{\sqrt{3}}{2} + 0 - \left(-1+\frac{1}{2}\right)\right] = \frac{\sqrt{3}-1}{2} - \frac{\pi}{12}$

23. $z = f(x,y) = 4 - x - 2y \geq 0$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$. So the solid is the region in the first octant which lies below the plane $z = 4 - x - 2y$ and above $[0,1]\times[0,1]$.



27. $V = \int_{-2}^2 \int_{-1}^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\,dy = 4\int_0^2 \int_0^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\,dy$

$= 4\int_0^2 \left[x - \frac{1}{12}x^3 - \frac{1}{9}y^2x\right]_{x=0}^{x=1} dy = 4\int_0^2 \left(\frac{11}{12} - \frac{1}{9}y^2\right) dy = 4\left[\frac{11}{12}y - \frac{1}{27}y^3\right]_0^2 = 4\cdot\frac{83}{54} = \frac{166}{27}$