2. $\int_0^1 \int_{2x}^2 (x-y)\,dy\,dx = \int_0^1 \left[xy - \frac{1}{2}y^2\right]_{y=2x}^{y=2} dx = \int_0^1 \left[x(2) - \frac{1}{2}(2)^2 - x(2x) + \frac{1}{2}(2x)^2\right] dx$

$= \int_0^1 (2x-2)\,dx = \left[x^2 - 2x\right]_0^1 = 1 - 2 - 0 + 0 = -1$

5. $\int_0^{\pi/2} \int_0^{\cos\theta} e^{\sin\theta}\,dr\,d\theta = \int_0^{\pi/2} \left[re^{\sin\theta}\right]_{r=0}^{r=\cos\theta} d\theta = \int_0^{\pi/2} (\cos\theta)\,e^{\sin\theta}\,d\theta = e^{\sin\theta}\Big]_0^{\pi/2} = e^{\sin(\pi/2)} - e^0 = e - 1$

8. $$\iint_D \frac{y}{x^5+1}\,dA = \int_0^1 \int_0^{x^2} \frac{y}{x^5+1}\,dy\,dx = \int_0^1 \frac{1}{x^5+1}\left[\frac{y^2}{2}\right]_{y=0}^{y=x^2} dx = \frac{1}{2}\int_0^1 \frac{x^4}{x^5+1}\,dx = \frac{1}{2}\left[\frac{1}{5}\ln\left|x^5+1\right|\right]_0^1$$

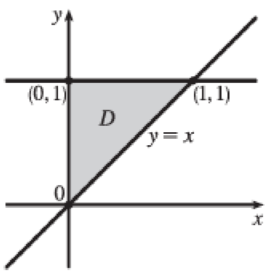
$= \frac{1}{10}(\ln 2 - \ln 1) = \frac{1}{10}\ln 2$

9. $\iint_D x\,dA = \int_0^\pi \int_0^{\sin x} x\,dy\,dx = \int_0^\pi \left[xy\right]_{y=0}^{y=\sin x} dx = \int_0^\pi x\sin x\,dx$ $\left[\begin{array}{c}\text{integrate by parts}\\ \text{with } u = x,\ dv = \sin x\,dx\end{array}\right]$

$= \left[-x\cos x + \sin x\right]_0^\pi = -\pi\cos\pi + \sin\pi + 0 - \sin 0 = \pi$

13. $\int_0^1 \int_0^{x^2} x\cos y\,dy\,dx = \int_0^1 \left[x\sin y\right]_{y=0}^{y=x^2} dx = \int_0^1 x\sin x^2\,dx = -\frac{1}{2}\cos x^2\Big]_0^1 = \frac{1}{2}(1-\cos 1)$

22.

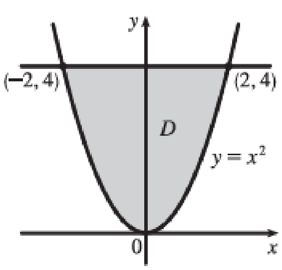


$V = \int_0^1 \int_x^1 (x^2 + 3y^2)\,dy\,dx$

$= \int_0^1 \left[x^2y + y^3\right]_{y=x}^{y=1} dx = \int_0^1 (x^2 + 1 - 2x^3)\,dx$

$= \left[\frac{1}{3}x^3 + x - \frac{1}{2}x^4\right]_0^1 = \frac{5}{6}$

25.

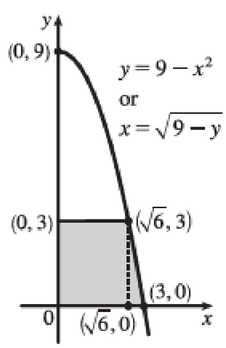


$V = \int_{-2}^2 \int_{x^2}^4 x^2\,dy\,dx$

$= \int_{-2}^2 x^2\left[\,y\,\right]_{y=x^2}^{y=4} dx = \int_{-2}^2 (4x^2 - x^4)\,dx$

$= \left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{-2}^2 = \frac{32}{3} - \frac{32}{5} + \frac{32}{3} - \frac{32}{5} = \frac{128}{15}$

42.



To reverse the order, we must break the region into two separate type I regions.

Because the region of integration is

$D = \left\{(x,y) \mid 0 \le x \le \sqrt{9-y},\ 0 \le y \le 3\right\}$

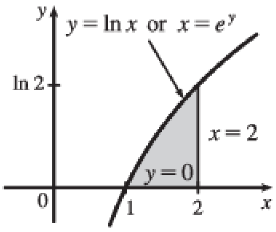
$= \left\{(x,y) \mid 0 \le y \le 3,\ 0 \le x \le \sqrt{6}\right\} \cup \left\{(x,y) \mid 0 \le y \le 9 - x^2,\ \sqrt{6} \le x \le 3\right\}$

we have

$$\int_0^3 \int_0^{\sqrt{9-y}} f(x,y)\,dx\,dy = \iint_D f(x,y)\,dA$$

$$= \int_0^{\sqrt{6}} \int_0^3 f(x,y)\,dy\,dx + \int_{\sqrt{6}}^3 \int_0^{9-x^2} f(x,y)\,dy\,dx$$

43.



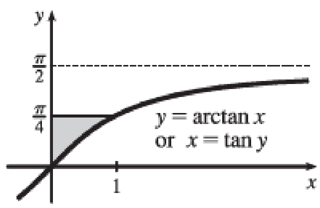
Because the region of integration is

$D = \{(x, y) \mid 0 \le y \le \ln x, 1 \le x \le 2\} = \{(x, y) \mid e^y \le x \le 2, 0 \le y \le \ln 2\}$

we have

$$\int_1^2 \int_0^{\ln x} f(x, y)\, dy\, dx = \iint_D f(x, y)\, dA = \int_0^{\ln 2} \int_{e^y}^{2} f(x, y)\, dx\, dy$$

44.



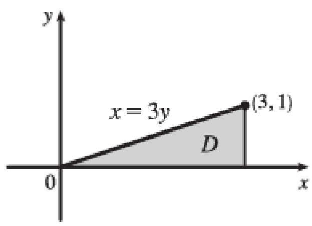
Because the region of integration is

$$D = \left\{(x, y) \mid \arctan x \le y \le \tfrac{\pi}{4}, 0 \le x \le 1\right\}$$
$$= \left\{(x, y) \mid 0 \le x \le \tan y, 0 \le y \le \tfrac{\pi}{4}\right\}$$

we have

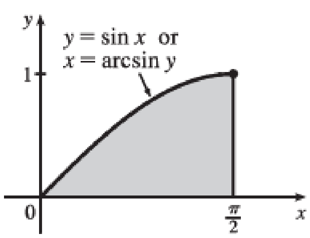
$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y)\, dy\, dx = \iint_D f(x, y)\, dA = \int_0^{\pi/4} \int_0^{\tan y} f(x, y)\, dx\, dy$$

45.



$$\int_0^1 \int_{3y}^{3} e^{x^2}\, dx\, dy = \int_0^3 \int_0^{x/3} e^{x^2}\, dy\, dx = \int_0^3 \left[e^{x^2} y\right]_{y=0}^{y=x/3} dx$$
$$= \int_0^3 \left(\frac{x}{3}\right) e^{x^2}\, dx = \tfrac{1}{6}\, e^{x^2}\Big]_0^3 = \frac{e^9 - 1}{6}$$

49.



$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$$
$$= \int_0^{\pi/2} \int_0^{\sin x} \cos x \sqrt{1 + \cos^2 x}\, dy\, dx$$
$$= \int_0^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, \left[y\right]_{y=0}^{y=\sin x}\, dx$$
$$= \int_0^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, \sin x\, dx \qquad \left[\begin{array}{c}\text{Let } u = \cos x,\ du = -\sin x\, dx, \\ dx = du/(-\sin x)\end{array}\right]$$
$$= \int_1^0 -u \sqrt{1 + u^2}\, du = -\tfrac{1}{3}\left(1 + u^2\right)^{3/2}\Big]_1^0$$
$$= \tfrac{1}{3}\left(\sqrt{8} - 1\right) = \tfrac{1}{3}\left(2\sqrt{2} - 1\right)$$