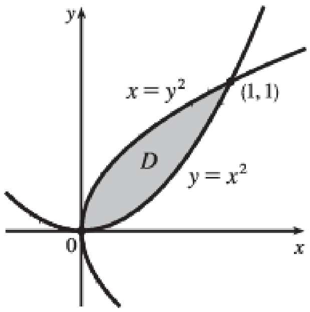
3. $m = \iint_D \rho(x,y)\,dA = \int_0^2 \int_{-1}^1 xy^2\,dy\,dx = \int_0^2 x\,dx \int_{-1}^1 y^2\,dy = \left[\frac{1}{2}x^2\right]_0^2 \left[\frac{1}{3}y^3\right]_{-1}^1 = 2 \cdot \frac{2}{3} = \frac{4}{3}$,

$\overline{x} = \frac{1}{m}\iint_D x\rho(x,y)\,dA = \frac{3}{4}\int_0^2 \int_{-1}^1 x^2y^2\,dy\,dx = \frac{3}{4}\int_0^2 x^2\,dx \int_{-1}^1 y^2\,dy = \frac{3}{4}\left[\frac{1}{3}x^3\right]_0^2 \left[\frac{1}{3}y^3\right]_{-1}^1 = \frac{3}{4}\cdot\frac{8}{3}\cdot\frac{2}{3} = \frac{4}{3}$,

$\overline{y} = \frac{1}{m}\iint_D y\rho(x,y)\,dA = \frac{3}{4}\int_0^2 \int_{-1}^1 xy^3\,dy\,dx = \frac{3}{4}\int_0^2 x\,dx \int_{-1}^1 y^3\,dy = \frac{3}{4}\left[\frac{1}{2}x^2\right]_0^2 \left[\frac{1}{4}y^4\right]_{-1}^1 = \frac{3}{4}\cdot 2 \cdot 0 = 0$.

Hence, $(\overline{x},\overline{y}) = \left(\frac{4}{3}, 0\right)$.

10.



$m = \int_0^1 \int_{x^2}^{\sqrt{x}} \sqrt{x}\,dy\,dx = \int_0^1 \sqrt{x}(\sqrt{x} - x^2)\,dx$

$= \int_0^1 (x - x^{5/2})\,dx = \left[\frac{1}{2}x^2 - \frac{2}{7}x^{7/2}\right]_0^1 = \frac{3}{14}$,

$M_y = \int_0^1 \int_{x^2}^{\sqrt{x}} x\sqrt{x}\,dy\,dx = \int_0^1 x\sqrt{x}(\sqrt{x} - x^2)dx = \int_0^1 (x^2 - x^{7/2})dx = \left[\frac{1}{3}x^3 - \frac{2}{9}x^{9/2}\right]_0^1 = \frac{1}{9}$

$M_x = \int_0^1 \int_{x^2}^{\sqrt{x}} y\sqrt{x}\,dy\,dx = \int_0^1 \sqrt{x}\cdot\frac{1}{2}(x - x^4)dx = \frac{1}{2}\int_0^1 (x^{3/2} - x^{9/2})dx$

$= \frac{1}{2}\left[\frac{2}{5}x^{5/2} - \frac{2}{11}x^{11/2}\right]_0^1 = \frac{1}{2}\cdot\frac{12}{55} = \frac{6}{55}$.

Hence $m = \frac{3}{14}$, $(\overline{x},\overline{y}) = \left(\frac{1/9}{3/14}, \frac{6/55}{3/14}\right) = \left(\frac{14}{27}, \frac{28}{55}\right)$.

12. $\rho(x,y) = k(x^2+y^2) = kr^2$, $m = \int_0^{\pi/2}\int_0^1 kr^3\,dr\,d\theta = \frac{\pi}{8}k$,

$M_y = \int_0^{\pi/2}\int_0^1 kr^4\cos\theta\,dr\,d\theta = \frac{1}{5}k\int_0^{\pi/2}\cos\theta\,d\theta = \frac{1}{5}k\left[\sin\theta\right]_0^{\pi/2} = \frac{1}{5}k$,

$M_x = \int_0^{\pi/2}\int_0^1 kr^4\sin\theta\,dr\,d\theta = \frac{1}{5}k\int_0^{\pi/2}\sin\theta\,d\theta = \frac{1}{5}k\left[-\cos\theta\right]_0^{\pi/2} = \frac{1}{5}k$.

Hence $(\overline{x},\overline{y}) = \left(\frac{8}{5\pi}, \frac{8}{5\pi}\right)$.

18. $I_x = \int_0^{\pi/2}\int_0^1 (r^2\sin^2\theta)(kr^2)\,r\,dr\,d\theta = \frac{1}{6}k\int_0^{\pi/2}\sin^2\theta\,d\theta = \frac{1}{6}k\left[\frac{1}{4}(2\theta - \sin 2\theta)\right]_0^{\pi/2} = \frac{\pi}{24}k$,

$I_y = \int_0^{\pi/2}\int_0^1 (r^2\cos^2\theta)(kr^2)\,r\,dr\,d\theta = \frac{1}{6}k\int_0^{\pi/6}\cos^2\theta\,d\theta = \frac{1}{6}k\left[\frac{1}{4}(2\theta + \sin 2\theta)\right]_0^{\pi/2} = \frac{\pi}{24}k$,

and $I_0 = I_x + I_y = \frac{\pi}{12}k$.