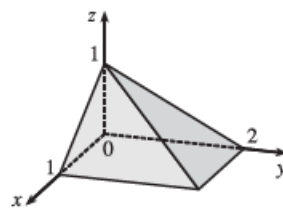


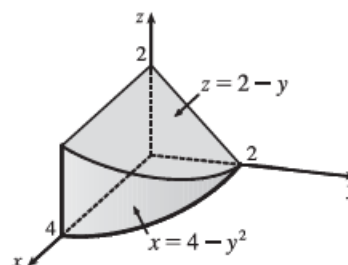
$$\begin{aligned}
 3. \int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz &= \int_0^1 \int_0^z [6xyz]_{y=0}^{y=x+z} \, dx \, dz = \int_0^1 \int_0^z 6xz(x+z) \, dx \, dz \\
 &= \int_0^1 [2x^3z + 3x^2z^2]_{x=0}^{x=z} \, dz = \int_0^1 (2z^4 + 3z^4) \, dz = \int_0^1 5z^4 \, dz = z^5 \Big|_0^1 = 1
 \end{aligned}$$

$$\begin{aligned}
 7. \int_0^{\pi/2} \int_0^y \int_0^x \cos(x+y+z) \, dz \, dx \, dy &= \int_0^{\pi/2} \int_0^y [\sin(x+y+z)]_{z=0}^{z=x} \, dx \, dy \\
 &= \int_0^{\pi/2} \int_0^y [\sin(2x+y) - \sin(x+y)] \, dx \, dy \\
 &= \int_0^{\pi/2} \left[-\frac{1}{2} \cos(2x+y) + \cos(x+y)\right]_{x=0}^{x=y} \, dy \\
 &= \int_0^{\pi/2} \left[-\frac{1}{2} \cos 3y + \cos 2y + \frac{1}{2} \cos y - \cos y\right] \, dy \\
 &= \left[-\frac{1}{6} \sin 3y + \frac{1}{2} \sin 2y - \frac{1}{2} \sin y\right]_0^{\pi/2} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}
 \end{aligned}$$

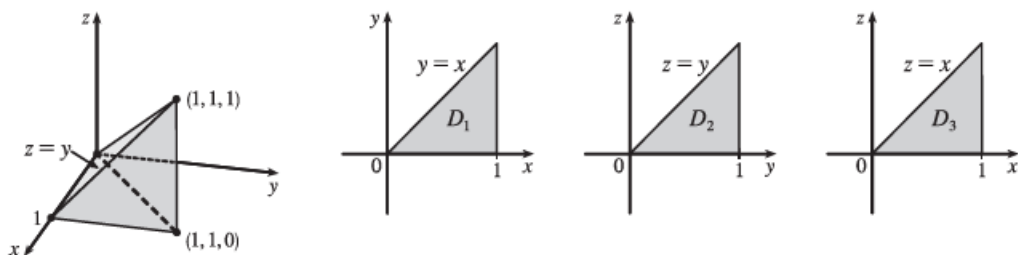
27. $E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq z \leq 1-x, 0 \leq y \leq 2-2z\}$,
 the solid bounded by the three coordinate planes and the planes
 $z = 1-x, y = 2-2z$.



28. $E = \{(x, y, z) \mid 0 \leq y \leq 2, 0 \leq z \leq 2-y, 0 \leq x \leq 4-y^2\}$,
 the solid bounded by the three coordinate planes, the plane $z = 2-y$,
 and the cylindrical surface $x = 4-y^2$.



35.



$$\int_0^1 \int_y^1 \int_0^y f(x, y, z) dz dx dy = \iiint_E f(x, y, z) dV \text{ where } E = \{(x, y, z) \mid 0 \leq z \leq y, y \leq x \leq 1, 0 \leq y \leq 1\}.$$

If D_1 , D_2 , and D_3 are the projections of E on the xy -, yz - and xz -planes then

$$D_1 = \{(x, y) \mid 0 \leq y \leq 1, y \leq x \leq 1\} = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x\},$$

$$D_2 = \{(y, z) \mid 0 \leq y \leq 1, 0 \leq z \leq y\} = \{(y, z) \mid 0 \leq z \leq 1, z \leq y \leq 1\}, \text{ and}$$

$$D_3 = \{(x, z) \mid 0 \leq x \leq 1, 0 \leq z \leq x\} = \{(x, z) \mid 0 \leq z \leq 1, z \leq x \leq 1\}.$$

Thus we also have

$$\begin{aligned} E &= \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, 0 \leq z \leq y\} = \{(x, y, z) \mid 0 \leq y \leq 1, 0 \leq z \leq y, y \leq x \leq 1\} \\ &= \{(x, y, z) \mid 0 \leq z \leq 1, z \leq y \leq 1, y \leq x \leq 1\} = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq z \leq x, z \leq y \leq x\} \\ &= \{(x, y, z) \mid 0 \leq z \leq 1, z \leq x \leq 1, z \leq y \leq x\}. \end{aligned}$$

Then

$$\begin{aligned} \int_0^1 \int_y^1 \int_0^y f(x, y, z) dz dx dy &= \int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx = \int_0^1 \int_0^y \int_y^1 f(x, y, z) dx dz dy \\ &= \int_0^1 \int_z^1 \int_y^1 f(x, y, z) dx dy dz = \int_0^1 \int_0^x \int_z^x f(x, y, z) dy dz dx \\ &= \int_0^1 \int_z^1 \int_z^x f(x, y, z) dy dx dz \end{aligned}$$