Summary. One dimensional Schrodinger Equation

1. The general Schrodinger equation is a postulate of QM.
2. Adopt the Hamiltonian $H$, for position, momentum degrees of freedom from Classical physics.
3. Using a plane wave for free particle (state of definite momentum and energy) a plausible form for the momentum operator in position representation is found.
4. With this operator form for KE = $p^2/2m$ write the Schrodinger equation as a second order partial differential equation in $x$ and $t$.
5. Separate the equation into the product of functions of $x$ and $t$.
6. Solve the time equation for energy eigenstates. The time dependence for an energy eigenstate is always of same form.
7. Now we have a second order ordinary differential equation in $x$.
8. The form of solution depends on the sign of $E-V(x)$.
   The forms are either real exponentials or sin and cos.
9. Continuity of derivative of wave function at a step discontinuity of potential.

Wells
10. Infinite well. The wave number ($k$) and energy are determined by the boundary conditions yielding a discrete Energy spectrum.
    Finite well: wave function in classically forbidden region.
11. Time dependence of the wave function if not in an energy eigenstate; Write as linear combination of energy eigenstates.
    Probability density = $\psi^*\psi$, expectation values of $x$, $H$, etc.

Tunneling
12. Example of a potential step; classically forbidden region.
13. Tunneling through a square potential barrier; normalizing to incoming flux;
14. Tunneling probability depends strongly on the thickness of barrier.
    Example: Scanning Tunneling Microscope

Double wells.
15. The states are nearly degenerate (depends on strength of barrier and separation of the wells).
16. The energy eigenstates are approximately the symmetric and antisymmetric combinations of single well states.
    Example: ammonia molecule.
17. If a particle starts out in one well it will oscillate between the wells.