In SI units the classical magnetic moment of a current loop is \( \mu = IA \) where \( A \) is the area of the loop of radius \( r \) and \( I \) is the current.

Then since the current is the charge divided by the period of the motion

\[
\mu = \frac{ev}{2\pi r} \pi r^2 = \frac{emvr}{2m} = \frac{eL}{2m} \frac{L}{\hbar} = \mu_B \frac{L}{\hbar}
\]

where \( L \) is angular momentum \( (= mvr) \), \( \mu_B \) is the Bohr Magneton, and \( \hbar \) is Planck’s constant divided by \( 2\pi \).

\[
\mu_B = \frac{e\hbar}{2m_e} = 5.8 \times 10^{-5} \text{ eV/Tesla} = 9.27 \times 10^{-24} \text{ J/Tesla}
\]

where \( e \) is the magnitude of the electron charge and \( m_e \) is the electron mass.

The units of \( \hbar \) are eV-seconds. Check that all the units make sense.

We similarly define the Nuclear Magneton using the proton mass.

\[
\mu_N = \frac{e\hbar}{2M_p}
\]

This classical expression can be taken over for the magnetic moment due to the intrinsic angular momentum or spin.

\[
\vec{M}_{\text{spin}} = -g_s \mu_B \frac{\vec{S}}{\hbar} = \gamma \vec{S}
\]

where \( g_s \) is called the “g factor” and is approximately 2 for electrons. It differs from 2 by about 1 part in 1000. The g factors for protons and neutrons are rather different due to their internal structure. \( \vec{S} \) is the spin angular momentum.

For electrons, protons, neutrons, and neutrinos the magnitude of the spin is \( \frac{1}{2} \) in units of \( \hbar \).

The gyromagnetic ratio \( \gamma \) is

\[
\gamma = -g_s \frac{\mu_B}{\hbar} \approx -\frac{2e\hbar}{2m_e \hbar} = -\frac{e}{m_e}
\]

The force on a magnetic moment is an inhomogeneous magnetic field is

\[
F_z = \mu \frac{\partial B}{\partial z}
\]