State Populations in Magnetic Field

The energy difference between the two energy levels of a magnetic moment of a spin 1/2 particle in a magnetic field $B_0$ is

$$\Delta E = \hbar \omega_0 \quad \omega_0 = -\gamma B_0 \quad \text{Assume } B_0 = 1 \text{ Tesla}$$

$$\Delta E = \frac{\hbar c}{\omega_0} \approx 42 \text{ MHz}$$

$$\Delta E \approx \frac{200 \text{ eV} - \text{nm}}{3 \times 10^8 \times 10^9} 2\pi \times 42 \times 10^6 \text{ Hz} \approx 1.8 \times 10^{-7} \text{ eV}$$

At room temperature $kT \approx 1/40 \text{ eV}$

$$\frac{\Delta E}{kT} \approx 40 \times 1.8 \times 10^{-7} \approx 7 \times 10^{-6}$$

The probability of a two state system being in a state with energy $E_0$ is

$$\frac{1}{Z} e^{-E_0/kT}$$

where $Z = e^{-E_0/kT} + e^{-E_1/kT} \quad \text{see Moore, Volume T, Example T6.2}$

The lower energy state (with the larger population) has energy $E_0$ and the higher, $E_1$.

The population difference divided by the sum of the populations is

$$\frac{P_0 - P_1}{P_0 + P_1} = \frac{e^{-E_0/kT} - e^{-E_1/kT}}{e^{-E_0/kT} + e^{-E_1/kT}} = 1 - e^{-(E_1 - E_0)/kT}$$

$$\approx 1 - e^{\Delta E/kT} \approx \frac{\Delta E}{2kT} \approx 3.5 \times 10^{-6}$$

Thus only about one in a million molecules contribute to the next magnetization of the sample. However there are of order $10^{23}$ molecules present so the signal is due to about $10^{17}$ magnetic moments.