Practice Problems III

1. A half-infinite well extends from \( x = 0 \) to \( x = L \). The potential energy within the well is zero. For \( x > L \) the potential energy is \( V \). The potential energy is infinite for \( x < 0 \).

   The energy of an electron in the ground state is \( V/2 \).

   a) Sketch the wave function
   b) Write the wave function and the matching conditions at the boundaries.
   c) Find the width of the well in terms of given quantities.
   d) Assume that \( V = 2 \) eV. Find the width of the well in nanometers.

2. For the problem of transmission through a potential barrier, use the expression for the transmission probability

\[
T = \frac{1}{1 + \frac{V^2}{4E(V-E)} \sinh^2(qa)}
\]

to find \( T \) in the limit of very small transmission probability.

   The Transmission probability is the magnitude squared of the outgoing wave divided by the magnitude squared of the incoming wave.

   \( q \) is the wave number inside the barrier. \( V \) is the height of the barrier and \( E \) is the incoming energy. \( a \) is the width of the barrier.

   The answer is

\[
T \approx \frac{16E(V - E)}{V^2} e^{-2qa}
\]

   showing that the transmission falls exponentially with barrier thickness.