The proton itself constitutes a magnetic dipole, though its dipole moment is much smaller than the electron's because of the mass in the denominator (Equation 6.60):

\[ \mu_p = \frac{g_p e}{2m_p} S_p, \quad \mu_e = -\frac{e}{m_e} S_e. \]  

[6.85]

(The proton is a composite structure, made up of three quarks, and its gyromagnetic ratio is not as simple as the electron's—hence the explicit \( g \)-factor \( g_p \), whose measured value is 5.59, as opposed to 2.00 for the electron.) According to classical electrodynamics, a dipole \( \mu \) sets up a magnetic field

\[ E_{\text{hf}}^1 = \frac{\mu_0 g_p e^2}{3\pi m_p \alpha^3} (S_p \cdot S_e). \]  

[6.89]

in the ground state. This is called spin-spin coupling, because it involves the dot product of two spins (contrast spin-orbit coupling, which involves \( S \cdot L \)).

In the presence of spin-spin coupling, the individual spin angular momenta are no longer conserved; the "good" states are eigenvectors of the total spin,

\[ S = S_e + S_p. \]  

[6.90]

As before, we square this out to get

\[ S_p \cdot S_e = \frac{1}{2} (S^2 - S_e^2 - S_p^2). \]  

[6.91]

\[ S_e^2 = \frac{\hbar^2}{2m_e c^2} \]

\[ S_p^2 = \frac{\hbar^2}{2m_p c^2} \]

\[ S^2 = \frac{\hbar^2}{2m \alpha^2} \]

**FIGURE 6.13:** Hyperfine splitting in the ground state of hydrogen.

The frequency of the photon emitted in a transition from the triplet to the singlet state is

\[ \nu = \frac{\Delta E}{h} = 1420 \text{ MHz}, \]  

[6.94]

and the corresponding wavelength is \( c/\nu = 21 \text{ cm} \), which falls in the microwave region. This famous 21-centimeter line is among the most pervasive and ubiquitous forms of radiation in the universe.