Problem 1.

In the well \( \psi_1(x) = A \sin kx \)

in the barrier \( \psi_2(x) = B e^{-qx} \)

From boundary conditions at \( x = L \)

\( A \sin kL = B e^{-qL} \) and \( kA \cos kL = -Bq e^{-qL} \)

Then \( \tan kL = -k/q \) and \( B = A \sin kL e^{qL} \)

If \( E = V/2 \) then \( k = \sqrt{\frac{2mV/2}{\hbar^2}} \) and \( q \) is the same.

\( k = q \) implies that \( \tan kL = -1 \) and therefore

\( kL = 3\pi/4 \) and \( L = \frac{3\pi}{4k} \)

\( V = 2 \text{ eV} \) and \( E = 1 \text{ eV} \). Then

\[
k = \sqrt{\frac{2mc^2(1 \text{ eV})}{(hc)^2}} = \sqrt{\frac{10^6}{200^2}} = 5 \text{ nm}^{-1}
\]

Then \( L = \frac{3\pi}{4k} = 0.47 \text{ nm} \).

Problem 2.

When \( qa >> 1 \) the transmission is small.

We have to approximate the hyperbolic sin for large \( qa \).

Then \( e^{-qa} \) is very small and the square of the hyperbolic sin

becomes \( \sinh^2 qa \approx \frac{1}{4} e^{2qa} \)

The approximate expression given in the problem then follows.

Problem 3 from HW VII.

This problem was worked out in detail in class.

In the well the solution is of the form \( \psi_1(x) = \sin kx \). (it must be 0 at \( x = 0 \).)

In the limiting case for a bound state the slope of this function at \( x = L \) is zero. Normally

the slope must be negative to match a falling exponential in the barrier region.

\( \sin kx \) having zero slope means that \( kL = \frac{\pi}{2} \)

In the limit of being “just” bound the energy \( E = V \) so that \( k = \sqrt{\frac{2mV}{\hbar^2}} \) and \( L = \frac{\pi}{2k} \)

As an example set \( V = 40 \text{ MeV} \) and the mass as 1000 MeV.

We then find that \( L \) is approximately \( 10^{-13} \text{ cm} \). (roughly the nuclear size).