Zone plate. We now come to Fresnel diffraction by a circular aperture. For simplicity, assume that the light comes from infinity. The wavefronts that pass through the aperture in Figure 2.3-21, therefore, are plane. We divide the aperture into half-period zones, which from zone to zone are $\frac{1}{2} \lambda$ farther away from point $P_0$. If $b$ is the distance from the center of the aperture to $P_0$, then $b + \frac{1}{2} \lambda$ corresponds to a (first) ring around the center of the aperture, $b + 2(\frac{1}{2} \lambda)$ to a second, wider ring, $b + 3(\frac{1}{2} \lambda)$ to the third ring, and $b + m(\frac{1}{2} \lambda)$ to the $m$th ring. These distances refer to boundaries between zones, rather than to the zones as such.

Fresnel's half-period zones on a plane wavefront.
\[ R_1 = \sqrt{(b + \frac{1}{2} \lambda)^2 - b^2} \]  

and the radius of the \( m \)th boundary 
\[ R_m = \sqrt{(b + \frac{1}{2} m \lambda)^2 - b^2} \]

Furthermore, the area of the first zone is 
\[ A_1 = \pi R_1^2 = \pi [(b + \frac{1}{2} \lambda)^2 - b^2] = \pi b \lambda + \frac{1}{4} \pi \lambda^2 \]

or, since \( \lambda \) is small compared with \( b \),
\[ A_1 = \pi b \lambda \]

The area of the second zone is 
\[ A_2 = \pi [(b + \lambda)^2 - b^2] - \pi b \lambda \approx 2 \pi b \lambda - \pi b \lambda = \pi b \lambda \]

and so on. The individual zones all have approximately the same area. Therefore, the contributions from these zones are very nearly equal; but since the zones differ by one-half of a wavelength, the resultant at \( R_0 \) is zero. However, any odd number of zones will give a bright center (while any even number will give a dark center).

If we now block out all odd-numbered zones, then only zones 2, 4, 6, \ldots will contribute to the light reaching \( R_0 \), and \( R_0 \) will be bright; it will become the focus of a zone plate* (Fig. 2.3-22).

\[ \frac{R_m^2}{R_1^2} = \frac{(b + \frac{1}{2} m \lambda)^2 - b^2}{(b + \frac{1}{2} \lambda)^2 - b^2} \]

Canceling \( b \) and \( \lambda \) gives
\[ R_m^2 + \frac{1}{2} R_m^2 \lambda = m (R_1^2 b + \frac{1}{2} R_1^2 \lambda) \]

and, since \( \lambda \ll R \),
\[ R_m = R_1 \sqrt{m} \]  

which means that the boundaries between zones have radii that are proportional to the square root of the natural numbers.

If we again square Equation [2.3-51] and rearrange the terms, we obtain
\[ b^2 = (b + \frac{1}{2} m \lambda)^2 - R_m^2 = b^2 + b \lambda + \frac{1}{2} b \lambda^2 - R_m^2 \]

and, since \( \lambda \ll b \),
\[ b \lambda = R_m^2 \]

The focal length, therefore, is
\[ f = b = \frac{R_m^2}{m \lambda} \]

Note the wavelength \( \lambda \) in this equation. It shows that a zone plate has severe chromatic aberration. (A conventional lens has chromatic aberration too, because its refractive index is a function of wavelength. This variation, though, is much smaller because \( n \) varies very slowly.)