**Fabry-Perot Interferometer**

The Fabry - Perot interferometer and related instruments make use of the multiple reflections between two plane parallel surfaces. They are all characterized by the same function, the Airy function $A(\phi)$, a function of increment of phase $\phi$ between successive beams. Suppose two parallel plates separated by an air gap of thickness $d$. Let the incident angle be $\theta$. For simplicity, refraction and absorption of light is disregarded. As shown, a fraction of the light $R_1$ will reflect at the first face and a fraction $T_1$ will be transmitted and this will then meet the second reflected beam where the amount $R_1 T_1$ will be reflected with a phase shift of $\phi/2$; ($\phi$ is the phase shift caused by 2 times the separation of plates) and $T_1 T_1 e^{j\phi/2}$ will be transmitted. The whole operation will repeat itself and net result is a series of diminishing geometric intensity. The total amplitude for $m$ waves is:

$$E_t = t_1 t_2 e^{j\phi/2} + t_1 t_2 r_1 r_2 e^{j3\phi/2} + t_1 t_2 r_1 r_2 e^{j5\phi/2} + \ldots + t_1 t_2 r_1 (m-1) r_2 (m-1)$$

$$= t_1 t_2 e^{j\phi/2} (1 + r_1 r_2 e^{j\phi} + r_1^2 r_2^2 e^{j2\phi} + \ldots)$$

$$E_t = t_1 t_2 e^{j\phi} \left( \frac{1}{1 - r_1 r_2 e^{j\phi}} \right)$$
Normalized transmission intensity therefore:

\[ I_t(\phi) = E_{t(total)} \times E_{t(total)}^* \]

\[ = \frac{|t_1 t_2|^2}{1 - |r'_1||r'_2|e^{-i\phi} - |r'_1||r'_2|e^{i\phi} + r'_1 r'_2} \]

\[ I_t = \frac{T_1 T_2}{1 - 2(R_1 R_2)^{0.5} \cos(\phi) + R_1 R_2} \]

Where

\[ T_1 = t_1^2; \]
\[ T_2 = t_2^2; \]
\[ R_1 = r_{11}^2; \]
\[ R_2 = r_{22}^2; \]
\[ t_1 = 1 - r_1 \]
\[ r_1 = -r_1^* \quad \text{(giving either direction from one mirror to the other is the same);} \]
\[ r_{11}^2 + t_1 t_1 = 1 \quad \text{(Stoke’s relation);} \]
\[ e^{i\phi} + e^{-i\phi} = 2\cos(\phi); \]
In the same manner, there is a series of reflection therefore, the sum of these reflection will be

\[ E_r = r_1 + (1 - r_1) \times (1 - r'_1) \times r'_2 \times e^{i\phi} + (1 - r_1) \times (1 - r'_1) \times r'_1 \times r'_2 \times e^{2i\phi} + \ldots \]

Since \( t_1 = 1 - r_1 \) and use above identities:

\[ = r_1 + (1 - r_1)^2 \times r_2 \times e^{i\phi} \times (1 + r_1 r_2 e^{i\phi} + \ldots) \]

\[ = r_1 + (1 - r_1)^2 r_2 e^{i\phi} \times \frac{1}{1 + (-r_1) r_2 e^{i\phi}} \]

Therefore

\[ E_{r\text{(total)}} = \frac{-r_1 + r_2 e^{i\phi}}{1 - r_1 r_2 e^{i\phi}} \]

\[ I_{r\text{(total)}} = \frac{|r_1|^2 - |r_1 r_2| e^{i\phi} - |r_1 r_2| e^{(-i)\phi} + |r_2|^2}{1 - |r_1 r_2| e^{i\phi} - |r_1 r_2| e^{(-i)\phi} + |r_1 r_2|^2} \]

\[ I_r = \frac{R_1 + R_2 - 2(R_1 R_2)^{0.5} \cos \phi}{1 + R_1 R_2 - 2(R_1 R_2)^{0.5} \cos \phi} \]

where

\[ \phi = 2 \times (2\pi \gamma n) / \lambda \times \cos(\theta) ; \]
Finesse $\xi$

Finesse $\xi$ is defined as the ratio of the half power bandwidth vs. the peak to peak full bandwidth in the transmission intensity curve.

(half of the transmission power) \[ \frac{I_t}{2} = \frac{T_1 T_2}{(1 + R_1 R_2 - 2(R_1 R_2)^{0.5}) \cos \left( \frac{\delta}{2} \right)} \]

where

\[ \delta = \text{half power bandwidth;} \]
\[ F = 4 \frac{(R_1 R_2)^{0.5}}{(1 - \sqrt{R_2 R_1})^2} \]

\[ \frac{1}{2} = \frac{1}{1 + F \sin \left( \frac{\delta^2}{2} \right)} \]

\[ F \sin \left( \frac{\delta}{2} \right) = 1 \]

Since $\delta$ is small therefore \[ \sin(\delta/2) = \delta/2 \]

\[ \frac{\delta}{2} = \frac{1}{\sqrt{F}} \]

in terms of angles:

\[ \xi = \frac{2\pi \sqrt{F}}{2} \]