Introduction to Nuclear Physics

Structure of Matter

Basic Nuclear Phenomenology

Nuclear Stability and Decay

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Course Website with slides, practice questions/answers:
http://depts.washington.edu/uwmip/

Structure of Matter

- Molecules: grouping of atoms
- Atoms:
  - Large sparse outer cloud: electron shells - chemistry
  - Small dense core: nucleus - nuclear physics

Nuclear Model

We can’t see the nucleus!

Treat as black box: typically probe it with particles and/or gamma rays - see what comes out and build a model

\[ p, n, d \rightarrow \text{Gamma rays} \]

\[ \text{particles} \rightarrow \text{Heaet, etc} \]

Nuclear Model: mathematical description that allows to calculate observed phenomena and to visualize the underlying processes.

Basic Constituents of Matter

- Nucleons:
  - Constituents of the nucleus:
    - Protons (charge +1)
    - Neutrons (charge 0)
  - Held together by the strong nuclear force
- Photons:
  - Transmit the electromagnetic force
  - Massless
  - A Gamma-ray is a photon produced in a nuclear reaction or decay
- Electrons, Positrons (Antielectrons)
  - Charge -1, +1
  - Interact via the electromagnetic and weak forces but not the strong nuclear forces
  - Nuclear $\gamma$ radiation consists of electrons (p) or positrons (p)

- $\nu$, $\bar{\nu}$ (Neutrinos, Antineutrinos)
  - approx. massless
  - weak interaction only

Classification of Atoms

Atom = Z electrons orbiting a nucleus
- with Z protons and N neutrons
- Nucleus: A nucleus (A = N + Z)
  - Z protons = 10^{-13} cm, 1.7 x 10^{-19} g (5.8 MeV/c^2)
  - N neutrons = 10^{-13} cm, 1.7 x 10^{-19} g (4.4 MeV/c^2)
- Z = Atomic Number = Number of Protons
- N = Neutron Number
- A = N + Z = Mass Number
- Atomic Size: \( \approx 10^{-10} \text{ cm} \)
- Atomic Size: \( \approx 10^{-13} \text{ cm} \)
- Atomic Mass: A atomic mass units (amu)
- 1 amu = 1.66 x 10^{-24} g (~ 931.5 MeV/c^2) = 1/12 of the mass of C-12
- Electron Mass: \( m_e = m_0 = 9.1 \times 10^{-24} \text{ g} (0.511 \text{ MeV/c}^2) \)

Examples of Atom/Nucleus Classification

- Notation:
  - Element (symbol X) with Z protons, N neutrons, A mass number:
    \( \rightarrow ^A_X\text{X} \)
- Example:
  - Fluorine: symbol F, atomic number 9, isotope with 18 nucleons (~ neutron number?)
    \( \rightarrow ^{18}_{\text{F}} \)
- Nuclides:
  - Nuclear species of atoms uniquely identified by number of protons, number of neutrons, and energy content of the nucleus.
  - Groups that share properties:
    - Isotopes: nuclides with the same proton (atomic) number, Z
    - Isobars: nuclides with the same mass number, A
    - Isomers: nuclides with the same A and Z, but different energy
**Factors in Nuclear Stability**

- Nuclear stability represents a balance between:
  - Nuclear "strong force" (basically attractive)
  - Electrostatic interaction (Coulomb force) between protons (repulsive)
  - Pauli exclusion principle
  - Residual interactions ("pairing force", etc.)

- Stability strongly favors N approximately equal to (but slightly larger than) Z. This results in the "band of stability" in the Chart of the Nuclides.

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**Phenomenology of Stability**

- Stability strongly favors nuclides with even numbers of protons and/or neutrons
  - ~50% are Even-Even
  - ~25% are Odd-even
  - ~25% are Even-Odd
  - Only 4 out of 266 stable nuclides are Odd-Odd! The heaviest stable Odd-Odd nuclide is °°N.

- "Magic Numbers" -- analogous to closed atomic shells
  - Result in many stable isotopes or isotones
  - Magic nuclei are particularly stable and more "inert"
  - Magic #s: 2, 8, 20, 28, 50, 82, 126

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**Chart of the Nuclides**

- Analogous to the Periodic Table of the elements
- Rows of constant Z (proton number): Isotopes (same chemical properties)
  - Example: C-12 and C-14, Z=6
- Columns of constant N: Isotones
  - Example: °°Tc (N=78) and °°Te (N=78)
- Isobars lie on diagonals of constant mass number A
  - Example: °°Tc and °°Te
- Isomers are the same entry with different energy levels
  - Example: °°Tc and °°Tc

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**Forces in the Nucleus**

- **Coulomb force**, $F_C \rightarrow \infty$ as $r \rightarrow 0$ (repulsive) increases with more protons
- **Nuclear strong force**, $F_{\text{nuc}}$ is short range (attractive) but VERY strong increases with more nucleons

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**Nuclear Binding and Stability**

- Protons and neutrons are more stable in a nucleus than free. The binding energy is the amount by which the nucleus' energy (i.e. mass) is reduced w.r.t. the combined energy (i.e. mass) of the nucleons.

  - **Example**: N-14 atom - Measured mass of N-14 = 14.0037
    - mass of 7 protons = 7 * (1.00727 amu) = 7.05089 amu
    - mass of 7 neutrons = 7 * (1.00866 amu) = 7.06062 amu
    - mass of 7 electrons = 7 * (0.00055 amu) = 0.00385 amu
    - mass of component particles of N-14 = 14.11536 amu

  - Binding energy is mass difference: $E_{\text{bind}} = 0.11229$ amu = 104.5 MeV
### Fundamental Concepts

- Total energy = $E = mc^2$
- Rest energy = $E_r = mc^2$
- Classic kinetic energy = $1/2(mv^2)$
- Classic momentum = $P = mv$
- Binding energy per nucleon = $E_b$ (total Binding E)/A

### More Raphex Questions

- **Raphex 2001, G 15.** The number of neutrons in a U-238 atom (Z=92) is:
  - A. 330
  - B. 258
  - C. 146
  - D. 92
  - E. Cannot tell from information given.

- **Raphex 2000, G 15.** Elements which have the same Z but different A are called:
  - A. Isotopes
  - B. Isomers
  - C. Isotones
  - D. Isobars
  - Isotopes have the same number of protons (atomic number, Z)

### Nuclear (Radioactive) Decays

- **Fission** -- only very heavy (high Z) elements (U, Pu, etc.) spontaneously fission. Nucleus splits into two smaller nuclei.
- **Alpha decay** -- like very asymmetric fission, usually occurs in heavy elements “above” the valley of stability. Nucleus emits an alpha particle: the same as a He nucleus, (2p 2n).
- **Beta decay** -- element X transforms into neighbor element X’. Nucleus converts a neutron to a proton or vice versa and emits a beta particle (electron): $n \rightarrow p + e^- + v$. Can also occur as Electron Capture.
- **Gamma decay** -- “excited” nucleus reduces its excitation energy without changing nuclear species (N, Z). Nucleus emits a gamma ray (electromagnetic quantum: the photon). Can also occur as Internal Conversion Electron.

### Raphex Questions

- **Raphex 2003, G 16.** In heavy nuclei such as $^{238}_{92}$U:
  - A. There are more protons than neutrons.
  - B. Protons and neutrons are equal in number.
  - C. There are more neutrons than protons.
  - D. Cannot tell from information given.

- **Raphex 2003, G 12.** A 10MeV ____ travels at the greatest speed in a vacuum.
  - A. Alpha particle
  - B. Neutron
  - C. Proton
  - D. Electron

### Nuclear Decay Occurs...

...when a nucleus is unstable

- An unstable nucleus metamorphoses (“decays”) into a more stable nucleus
- Difference in energy levels $\Rightarrow$ mass and kinetic energy of the decay products
- Mass is converted into energy $\Rightarrow$ radiation: $E = mc^2$

### Alpha Decay

- Spontaneous emission of an $\alpha$ particle (2p 2n = He-4 nucleus)
- Only occurs with heavy nuclides (A>150)
- [often followed by gamma and characteristic x-ray emission]
- Emitted with discrete energy (nuclide-dependent, 2-10 MeV)
- Not used in medical imaging

\[
^{\alpha}X \rightarrow ^{A+\alpha}_{Z+2}Y + ^2He^2 + \text{transition energy}
\]

Example: $^{220}_{86}$Rn $\rightarrow ^{216}_{84}$Po $+ ^3He^2 + 6.4$ MeV transition energy
Beta (β) Decay

**Basis:**
- A free neutron decays: neutron $\rightarrow$ proton + electron + antineutrino
- Half-life ($T_{1/2}$) = 10.5 minutes (for a free (unbound) neutron)
- The released energy is split between 3 decay products, so each has a spectrum of possible energies up to the max
- This basic process (and its inverse) forms the basis of all β decay

$$\text{Electron (beta, negatron)} \rightarrow \text{Neutron} \rightarrow \text{Proton}$$

**Gamma Decay (Isomeric Transition)**
- Nucleus in excited state with lower-lying nuclear energy levels open (usually formed as product daughter of other decay)
  - Excited state marked by * (e.g. 99mTc)
- Gamma ray (high-energy photon) emitted during transition to stable state
  - Usually occurs instantaneously
  - Some excited states persist longer (10^{-12} sec - 600 years!)
  - Metastable or isomeric state (e.g. 99mTc)
- Can also emit internal conversion electron - all energy is transferred to inner shell electron, which is ejected, characteristic x-rays follow to fill the opening

**Energy Level Diagram: Positron decay**
- Beta+ 97%
- EC 3%

$p = 1.67252 \times 10^{-27}$ kg

$N = 1.67482 \times 10^{-27}$ kg

$E = 0.0009 \times 10^{-27}$ kg

Neutrino mass = 0

So, part of Energy -> mass

**Energy Level Diagram**
- Nuclear Medicine Example: $^{43}$Tc$^{99m}$
- 82%
- 141 keV γ
- 141 γ decay $T_{1/2} = 6$ hours
- 0.143 Electron shell transition
- 0.143 Neutrino mass
- 0.0

**Beta (β) Decay - II**
- Free neutron decay: $n \rightarrow p + e^- + \bar{\nu}$
- Beta (β) emission: $^A_ZX \rightarrow ^{A-1}_{Z-1}Y + e^-$
- Positron (β⁺) emission: $p \rightarrow n + e^+ + \nu$
- Electron (e⁻) capture: $p + e^- \rightarrow n + \nu$
- Orbital electron captured, characteristic x-ray emission follows
Decay Terms

- Activity, $A$
  - Number of radioactive decays per unit time ($t$) - or
  - Change in number of radioactive nuclei present: $A = -dN/dt$
  - Depends on number of nuclei present. During decay of a fixed initial number of nuclei, $A$ will decrease.
  - Measured in Becquerel (Bq):
    - $1$ Bq = $1$ disintegration per second (dps)
    - $1$ Ci = $3.7 	imes 10^{10}$ Bq ($1mCi = 37$ MBq)

- Decay Constant, $\lambda$
  - Fraction of nuclei that will decay per unit time: $\lambda = (-dN/dt)/N = A/N$
  - Related to activity: $A = \lambda N$
  - Constant in time, characteristic of each nuclide
  - Example: Tc-99m has $\lambda = 0.1151$ hr$^{-1}$, i.e. $11.5\%$ decay per hour
  - Mo-99 has $\lambda = 0.252$ day$^{-1}$, i.e. $25.2\%$ decay per day

Fundamental Decay Equation

$$N_t = N_0 e^{-\lambda t}$$

$$A_t = A_0 e^{-\lambda t}$$

Example: Patient injected with $10$ mCi F-18 FDG, scan started 60 min later. How much activity is present in the scan?

$$A(60\text{min}) = A_0 x e^{-\lambda t} = 10 \text{mCi} \times e^{-0.0063\text{min}^{-1} \times 60 \text{min}}$$

Nuclear decay is a statistical process - can only predict averages.

Decay Terms - II

- Half-life, $T_{1/2}$
  - Time after which half of the initially present nuclei ($N_0$) will have decayed
  - After $n$ half-lives, $N = N_0 \times (1/2)^n$ nuclei will be left
  - Also characteristic of nuclide, constant in time
  - Related to decay constant, $\lambda$, by natural log of $2$:
    - $\lambda = \ln 2 / T_{1/2} = 0.693 / T_{1/2}$

Raphex Questions

- Raphex 2003 G 28. The following radioactive transformation represents $\gamma$-ray decay:
  - $\lambda X \rightarrow \lambda Z Y + \gamma + \nu$
  - A. Alpha
  - B. Beta minus
  - C. Beta plus
  - D. Electron capture
  - E. Isomeric transition

Answer: D - As $Z$ decreases by $1$, it must be either beta plus or electron capture. However, no positron is created, so beta plus is ruled out.

Extra: Models of the Nucleus

- Liquid Drop model
- Shell model
- Optical model
- Collective model (includes ‘modern’ notions of string vibration states, etc).

⇒ The one of interest to Nuclear Medicine is the Shell model
⇒ It need to explain nuclear stability and decay
Shell model

- Similar to the electron shell model in atoms
  - “Magic numbers”
- Complicated by two kinds of nucleons (proton, neutron)

Consider $^{24}\text{Mg}$

$^{24}\text{Na} \rightarrow p + e^- + \nu$

$^{24}\text{Mg}$

Free

Bound

Energy

$p$ $n$

Ground state

Where does the energy go?

- When the nucleon changes levels (but not species), the energy is usually emitted as a gamma ray (or internal conversion electron).

$^{24}\text{Na}$

$^{24}\text{Mg}$

2.76 MeV gamma ray

1.36 MeV gamma ray

$^{24}\text{Na}$

$n \rightarrow p + e^- + \nu$

Beta decay

Let's recap a few points
Nuclear Decay Occurs...

...when a nucleus is unstable (lower open energy levels)

- An unstable nucleus metamorphoses ("decays") into a more stable (more tightly bound) nucleus
- Difference in binding energy $\rightarrow$ mass and kinetic energy of the decay products
- Mass is converted into energy $\rightarrow$ radiation

\[ E = mc^2 \]

Nuclear Decay Characteristics

- Type of decay (fission, alpha, beta, electron capture, etc.)
- Decay constant (transformation rate)
  \[ N = N_0 e^{-\lambda t} \]
  \[ T_{1/2} = \frac{0.693}{\lambda} \]
- Radiation type ($\beta^+$, $\beta^-$, $\alpha$, fission fragments, etc.)
- Emission energy -- if continuum, then express as maximum energy or mean (average) energy
- Associated gamma ($\gamma$) or x rays
- "Daughter nucleus"
  - is it stable?
  - Produced in "ground state" or "excited state"?
  - With what probabilities ("branching ratios")?

What's next

Next week we will take a look at
Radiation detection and measurements
Dr. Lawrence MacDonald