Warm-up. If the position of an object is given by \( s(t) = \cos(\pi t/4) \) for \( 0 \leq t \leq 10 \). Find its velocity, speed, and acceleration at time \( t \).

Moving through space works the same way except position, velocity, and acceleration become vector quantities.

<table>
<thead>
<tr>
<th>Motion in a plane</th>
<th>Motion through space</th>
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</thead>
<tbody>
<tr>
<td>( s(t) ) is position</td>
<td>( \mathbf{r}(t) ) is position</td>
</tr>
<tr>
<td>( s'(t) = v(t) ) is velocity</td>
<td>( \mathbf{r}'(t) = \mathbf{v}(t) ) is velocity</td>
</tr>
<tr>
<td>( s''(t) = v'(t) = a(t) ) is acceleration</td>
<td>( \mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t) ) is acceleration</td>
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Example Find the velocity, speed and acceleration of a particle with the given position function.

\[ \mathbf{r}_1(t) = t \mathbf{i} + t^2 \mathbf{j} + 2 \mathbf{k} \]
\[ \mathbf{r}_2(t) = t \sin t \mathbf{i} + t \cos t \mathbf{j} + t^3 \mathbf{k} \]

Of course, we can use integrals of vector functions to work backwards and find a position vector given information about velocity (or acceleration.)

Problem. Find the velocity and position vectors for a particle with acceleration \( \mathbf{a}(t) = 2 \mathbf{i} + 6t \mathbf{j} + 12t^2 \mathbf{k} \) provided \( \mathbf{v}(0) = \mathbf{i} \) and \( \mathbf{r}(0) = \mathbf{j} - \mathbf{k} \).
All of mechanics can be filtered through the lens of vector functions. \( \mathbf{F}(t) = m\mathbf{a}(t) \).

**Problems.**

1. What force is required so that a particle of mass \( m \) has the position function \( \mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k} \)?

2. A force with magnitude 20 N acts directly upward from the \( xy \)-plane on an object with mass 4 kg. The object starts at the origin with initial velocity \( \mathbf{v}(0) = \mathbf{i} - \mathbf{j} \). Find its position function and its speed at time \( A \) projectile is fired with an initial speed of 500 m/s and angle of elevation 30°. Find (a) the range of the projectile, (b) the maximum height reached, and (c) the speed at impact.

Sometimes it is useful to resolve the acceleration vector into two components

- one in the direction of motion (i.e. in the direction of the tangent vector)
- one in the direction of the normal.

Recall that \( \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \)

So \( \mathbf{v} = \) Now differentiate with respect to \( t \).

This leads to \( \mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N} \).

**Example.** Find the tangential and normal components of the acceleration vector \( \mathbf{r}(t) = e^t\mathbf{i} + \sqrt{2}t\mathbf{j} + e^{-t}\mathbf{k} \).