Precise Control of Cure Processes During Repair

A. F. Emery, UW
Eric Casterline, HEATCON
Optimization of Composite Repairs

**GOAL:** Produce a Spatially Uniform Temperature at the Repair Site for a Specified Time

**APPROACH:** Custom Design a Heating Blanket or Heating Source with Spatially Varying Heating Density

**PROBLEM:** How to Determine the Needed Distribution of Heat
Schematic of Heat Loss

What are $Q_1$ and $Q_2$
Required: The Thermal Characteristics of the Repair Site
Proposed: Heat the system using a Thermal Blanket with Constant Heating Density and Measure Temperatures—Estimate Thermal Characteristics
Inverse Method

Using measured temperatures estimate the characteristics, $P_i$

$$T(t) = M(x,y,z,t,P_1,P_2,...,P_n)$$

by choosing values of $P_i$ until the model gives a good fit to the data

$$\begin{bmatrix}
\Delta P_1 \\
\Delta P_2 \\
\Delta P_n
\end{bmatrix} = (A^T A)^{-1} A^T \{T - M(P_0)\}$$

where

$$A = \begin{bmatrix}
\frac{\partial \{M\}}{\partial P_1} & \frac{\partial \{M\}}{\partial P_2} & \frac{\partial \{M\}}{\partial P_n}
\end{bmatrix}$$
Example

\[ T(x,0) = 100 \]

\[ q = h(T(L,t) - T_\infty) \]

Estimate conductivity \( (k) \) and heat transfer coefficient \( (h) \) using Temperatures measured at random points, \( x_i \), at several values of time.
The Measured Temperatures are noisy
Can we estimate $k$ by itself?

Can we estimate $h$ by itself?
Estimating $k$ and $h$ simultaneously
Estimating $k$ and $h$ with multiple sensors

The Problem is that each search requires 4 calculations of the temperatures
Using a Reduced Model

1) Compute the Model response for a range of Parameters
2) Extract a set of patterns that accurately reflect the response
3) Expand the solution in terms of these patterns

Why?

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Number of Parameters</th>
<th>Assembly</th>
<th>Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>N~30</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2D</td>
<td>N~900</td>
<td>30</td>
<td>27 \times 10^3</td>
</tr>
<tr>
<td>3D</td>
<td>N~27000</td>
<td>900</td>
<td>700 \times 10^6</td>
</tr>
</tbody>
</table>

Process:
POD   Proper Orthogonal Decomposition

1) Each pattern is a vector of nodal values of the Temperature
2) Each pattern vector is orthogonal to all others
3) Each subsequent pattern contains less information
4) The solution is a linear combination of the pattern vectors

Difficulties
1) No proof that the solution using a reduced model approaches the true response
2) Solutions can be obtained only for parameters within the range of the sampled responses

Advantage
1) If M patterns are used, the problem is equivalent to N=M
2) For nonlinear problems, FEM matrices must be inverted for each choice of parameter, but again N=M
Fourier Series Solution

\[ T(x, t) = \sum_{\beta} f(x, \beta) e^{-\beta^2 kt / \rho c} \]
Reproducing the Exact Solution with POD

Interpolating for $k$, $h$, and $\rho c$

$k = 5.600e-001$, $h = 2.000e-001$, $\rho c = 1.955e+001$

rms error $1.441e-004$

Number of Vectors 4
Final Procedure

1) Run 3D FEM for ranges of all parameters to be estimated

2) Extract Patterns

3) Generate Sensitivities from the Reduced Model

4) Estimate Parameters

5) Predict Heat Losses from Repair Site

6) Design Heating Blanket