UW Biomechanics Symposium
April 2001

An Investigation Of Global Coordination Of Gaits In Animals With Simple Neurological Systems Using Coupled Oscillators.

Lesley Ann Low and Per G. Reinhall
Department of Mechanical Engineering
University of Washington
Seattle, Washington 98195 USA

Introduction

Limit cycle oscillators arise in many biological systems such as cell division [4] and the neural networks in the spine and brain that control rhythmic actions such as breathing and chewing [3]. Recently, research has shifted towards the study of the behavior of a system of coupled oscillators, with the main focus on synchronization criteria. The behavior of several coupled limit cycle oscillators display a significantly wider range of phenomenon than a single limit cycle oscillator and can be used to model systems such as neural synapses of a swimming fish [2]. The different gaits found in the systems of coupled oscillators can be directly applied to modeling locomotion of such biological systems as swimming fish and swimming jellyfish.

Experimentation performed on lamprey show that the central pattern generator controls locomotion and there is a constant phase lag between the impulses in the segments of the spine [2]. The central pattern generator can be modeled with a system of limit cycle oscillators each coupled to its nearest neighbor [4,5]. Also, Anderson [1] has argued that the neurons that control locomotion of at least one type of jellyfish (*Polyorchis penicillatus*) are coupled together and display behavior indicative of different combinations of synchronously firing coupled neurons.

Methodology

The van der Pol equation has been adopted as a common mathematical model for limit cycle oscillators [6] and its behavior is well understood. Rand and Holmes [6] first formulated and studied the problem of a pair of van der Pol oscillators with weak linear diffusive coupling:

$$\ddot{x}_i + e(x_i^2 - 1)\dot{x}_i + (1 + e\Delta)x_i = \ldots$$
$$e\sum A_{ji}(x_j - x_i) + e\sum B_{ji}(\dot{x}_j - \dot{x}_i)$$

(the left hand side being the van der Pol equation), where the dot represents differentiation with time, *x* and *y* are the dependent variables, eA and eB are the coupling parameters, and e and ? the non-linearity and detuning parameters, respectively.

The equations above are used to define systems with three different coupling schemes:
1) Nearest neighbor coupling in a ring (where the n^{th} oscillator is coupled to the 1^{st} oscillator):

$$i = 1..n,\ j = \begin{cases} i+1, n & i = 1 \\ i+1, i-1 & i = 2..n-1 \\ 1, i-1 & i = n \end{cases}$$

2) Nearest neighbor coupling in a chain:

$$i = 1..n,\ j = \begin{cases} i+1 & i = 1 \\ i+1, i-1 & i = 2..n-1 \\ i-1 & i = n \end{cases}$$

3) Each oscillator coupled to each oscillator:

$$i = 1..n, \quad j = 1..n, \quad j \neq i$$

The term “diffusive" coupling is inspired by the field of biology. The coupling of two cells is described by the diffusion of the solute concentrations between the cells [6]. Thus, the coupling is written as the difference in the concentrations (*x*-*y*). Within each cell there are inhibitors (negative direction of solute flow) and promoters (positive direction of solute flow) that determine the 'direction' of the solute flow. Because there is both positive and negative direction of solute flow it follows that the coupling can be positive and negative. In addition to the diffusive coupling (*x*-*y*), the coupling also includes the linear difference between the rates of the oscillators.

These systems are studied to determine the stability (on the coupling parameter space) of different types of gaits.

Results

Using numerical simulations (Matlab Simulink), the stability regions and dynamical behavior of the coupled systems is investigated. With three or more oscillators each coupling scheme displays a wide range of phase-locked modes (in-phase, out-of-phase and shifted phase-locked).

The most interesting gait is where each oscillator is a fixed phase-lag from its nearest neighbor and these phase-lags are equal. Therefore, the beating of the system can be characterized as a wave, starting at oscillator 1, then moving to oscillator 2 (a nearest neighbor), continuing through the system to n^{th} oscillator, where the 'wave' begins again at oscillator 1. This behavior is found in systems of 3 and 5 oscillators with nearest neighbor coupling in a ring.

Another phase-locked mode investigated is that in which two or more oscillators beat together and are a fixed phase-lag apart from another group (2 or more) of beating oscillators. Physically, this is where one side of a ring beats together and is shifted from the other side of the ring. This behavior is found in all systems with four or more oscillators.

Conclusion

The gaits described above show promise in the ability to model 1) the central pattern generator for a swimming fish using the concept of a wave propagating down the spine and 2) the swimming behavior of jellyfish, including their ability to turn.

References

[1] Anderson, P.A.V., Mackie, G.O., Electrically Coupled, Photosensitive Neurons Control Swimming in a Jellyfish, *Science*, 197, 186-189.

[2] Cohen, A.H., Holmes, P.J., and Rand, R.H., 1982, The nature of the coupling between segmental oscillators of the lamprey spinal generator for locomotion: A mathematical model, *Journal of Mathematical Biology*, 13, 345-369.

[3] Foo, S.Y., 1994, Coupled Nonlinear Oscillators in Biological Systems.

[4] Murray, J.D., 1989, *Mathematical Biology*.

[5] Rabinovich, M., Selverston, A., Rubchinsky, L., and Huerta, R., 1996, Dynamics and Kinematics of Simple Neural Systems, *Chaos*, 6(3), 288-296.

[6] Rand, R.H., and Holmes, P.J., 1980, Bifurcation of Periodic Motions in Two Weakly Coupled van der Pol Oscillators, International Journal of Non-Linear Mechanics, 15, 387-399.