Introduction to Optimization Models

EXCEL Modeling of Simple **Linear** Problems

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Nuts and bolts of optimization models

- Decision variables
- Parameters (data)
- Constraints
- Performance objective
- Linear problems = constraints and performance objective are linear functions of decision variables

Problem 1: Diet problem

- To decide the quantities of different food items to consume every day to meet the daily requirement (DR) of several nutrients at minimum cost.

Diet problem continued

- What decision variables do we need?
  - Units of wheat consumed every day X
  - Units of rye consumed every day Y
- What type of data do we need?
- What constraints do we need?
- What is our objective function?

Diet problem data

	Wheat	Rye	DR
Carbs/unit	5	7	20
Proteins/unit	4	2	15
Vitamins/unit	2	1	3
Cost/unit	0.6	0.35	

$$\text{minimize} \quad 0.6X + 0.35Y$$

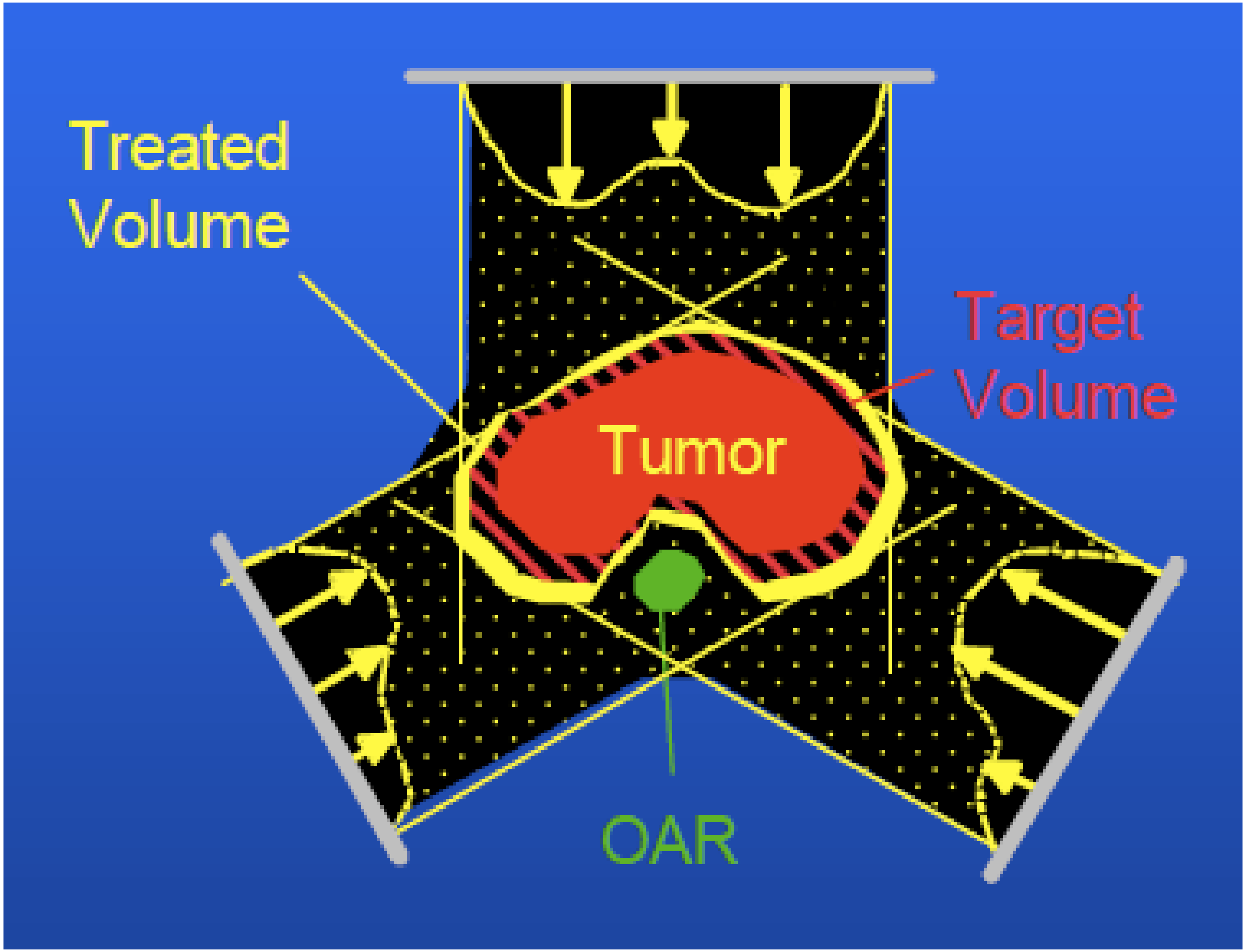
subject to

$$5X + 7Y \geq 20$$
$$4X + 2Y \geq 15$$
$$2X + Y \geq 3$$
$$X \geq 0$$
$$Y \geq 0$$

Optimal solution : X = 3.611 Y = 0.278
Cost : 2.2639

Problem 2 : Cancer treatment with radiation therapy

- One possible way to treat cancer is radiation therapy
  - An external beam treatment machine is used to pass radiation through the patient’s body
  - Damages both cancerous and healthy tissues
  - Typically multiple beams of different dose strengths are used from different sides and different angles
- Decide what beam dose strengths to use to achieve sufficient tumor damage but limit damage to healthy tissues


Treated Volume
Target Volume
Tumor
OAR

Radiation therapy data

	Fraction of dose absorbed by area (average)		
Area	Beam 1	Beam 2	Restriction on total average dose
Healthy anatomy	0.4	0.5	Minimize
Critical tissue	0.3	0.1	at most 2.7
Tumor region	0.5	0.5	equal to 6
Center of tumor	0.6	0.4	at least 6

Decision variables x_1 and x_2 represent the dose strength for beam 1 and beam 2 respectively minimize $0.4x_1 + 0.5x_2$ ← dose to healthy anatomy

subject to

$0.3x_1 + 0.1x_2 \leq 2.7$ ← dose to critical tissue

$0.5x_1 + 0.5x_2 = 6$ ← dose to tumor

$0.6x_1 + 0.4x_2 \geq 6$ ← dose to tumor center

$x_1 \geq 0$

$x_2 \geq 0$

Optimal solution : $x_1 = 7.5$ $x_2 = 4.5$

Dose to healthy anatomy : 5.25

Problem 3 : Transportation problem

- A non-profit organization manages three warehouses and four healthcare centers. The organization has estimated the requirements for a specific vaccine at each healthcare center in number of boxes of vials. The organization knows the number of boxes of vials available at each warehouse. They want to decide how many boxes of vials to ship from the warehouses to the healthcare centers so as to meet the demand for the vaccine at minimum total shipping cost.
- Decision variables X_{ij} the number of boxes shipped from warehouse i to healthcare center j.

Transportation problem data

	HC1	HC2	HC3	HC4	AVAILABILITY
W1	464	513	654	867	75
W2	352	416	690	791	125
W3	995	682	388	685	100
REQUIREMENT	80	65	70	85	300

$$\text{minimize} \quad 464X_{11} + 513X_{12} + 654X_{13} + 867X_{14} + 352X_{21} + 416X_{22} + 690X_{23} + 791X_{24} + 995X_{31} + 682X_{32} + 388X_{33} + 685X_{34}$$

total shipping cost

subject to

warehouse material balance

$$X_{11} + X_{12} + X_{13} + X_{14} = 75$$
$$X_{21} + X_{22} + X_{23} + X_{24} = 125$$
$$X_{31} + X_{32} + X_{33} + X_{34} = 100$$

boxes available must be shipped

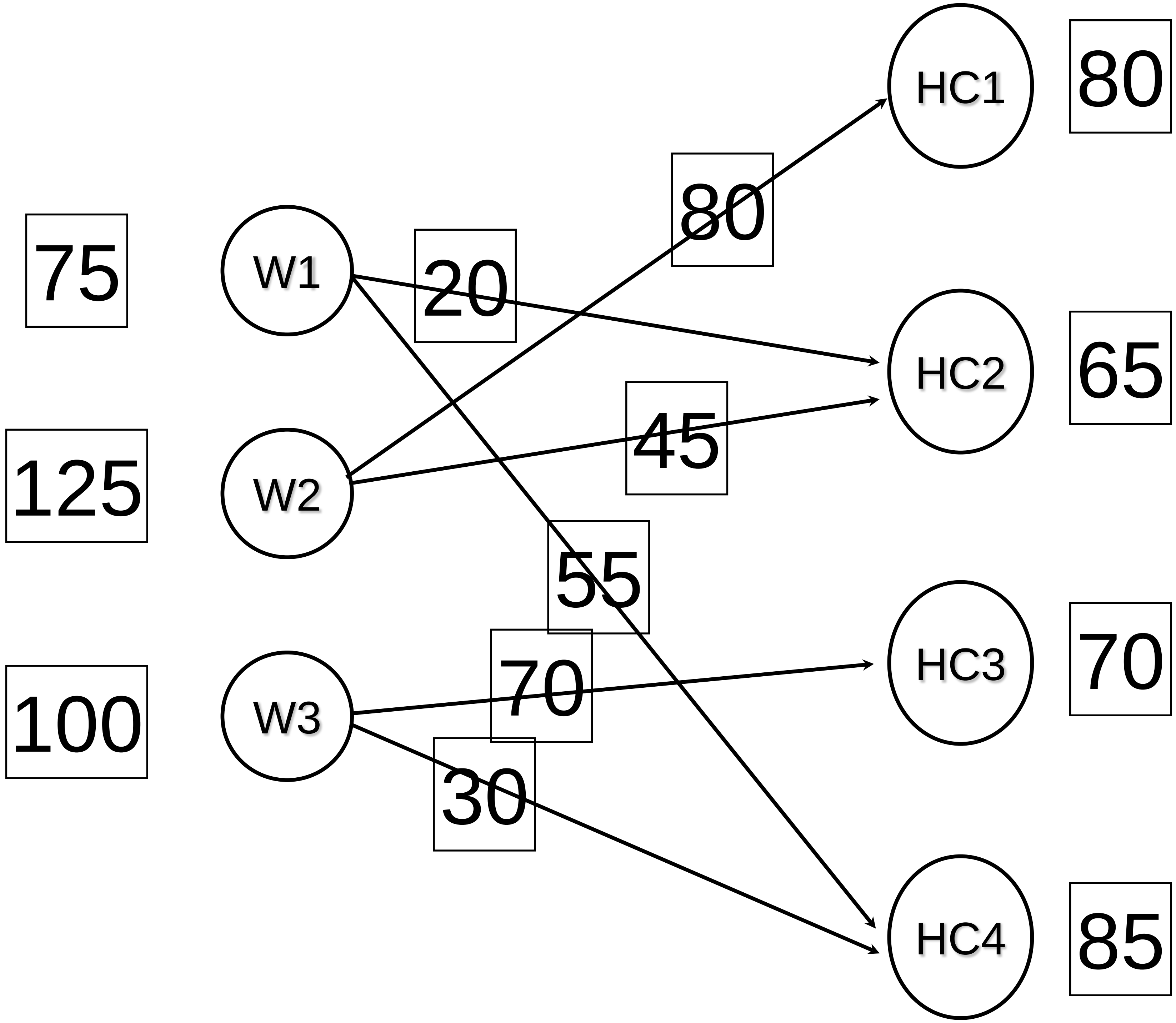
health center material balance

$$X_{11} + X_{21} + X_{31} = 80$$
$$X_{12} + X_{22} + X_{32} = 65$$
$$X_{13} + X_{23} + X_{33} = 70$$
$$X_{14} + X_{24} + X_{34} = 85$$

demand for boxes must be met

$$X_{ij} \geq 0, \quad \text{for } i=1,2,3 \text{ and } j=1,2,3,4$$

Solution



Cost : 152535

Problem 4 : Air pollution problem

- A steel plant has been ordered to reduce its emission of 3 air pollutants - particulates, sulfur oxides, and hydrocarbons
- The plant uses 2 furnaces
- The plant is considering 3 methods for achieving pollution reductions - taller smokestacks, filters, better fuels
- The 3 methods are expensive, so the plant managers want to decide what combination of the 3 to employ to minimize costs and yet achieve the required emission reduction.

Pollutant	Required emission reduction (million pounds per year)
Particulates	60
Sulfur oxides	150
Hydrocarbons	125

Emission reduction (million pounds per year) if the method is employed at the highest possible level						
Pollutant	Taller Smokestacks		Filters		Better Fuels	
	F1	F2	F1	F2	F1	F2
Particulates	12	9	25	20	17	13
Sulfur oxides	35	42	18	31	56	49
Hydrocarbons	37	53	28	24	29	20

Annual cost of employing a method at the highest possible level (million dollars)		
Method	F1	F2
Taller smokestacks	8	10
Filters	7	6
Better fuels	11	9

Decision variables - fraction of the highest possible level of a method employed		
Method	F1	F2
Taller smokestacks	x_1	x_2
Filters	x_3	x_4
Better fuels	x_5	x_6

$$\text{minimize} \quad 8x_1 + 10x_2 + 7x_3 + 6x_4 + 11x_5 + 9x_6$$

total cost

subject to

$$12x_1 + 9x_2 + 25x_3 + 20x_4 + 17x_5 + 13x_6 \geq 60$$
$$35x_1 + 42x_2 + 18x_3 + 31x_4 + 56x_5 + 49x_6 \geq 150$$
$$37x_1 + 53x_2 + 28x_3 + 24x_4 + 29x_5 + 20x_6 \geq 125$$

emission reduction requirements cost

$$x_j \leq 1, \text{ for } j=1,2,3,4,5,6$$
$$x_j \geq 0, \text{ for } j=1,2,3,4,5,6$$

Optimal solution : $(x_1, x_2, x_3, x_4, x_5, x_6)$ = (1,0.623,0.343,1,0.048,1)

Cost : 32.16 million dollars

Reference

"Introduction to Operations Research" by Hillier and Lieberman, 9th edition, McGraw-Hill