Targeted Adaptive Mesh Refinement for Tsunami Modeling Using Adjoint Equations

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Objectives

Tsunami modeling often combines the need for an ocean-wide simulation with the requirement that a small region of the coast (some community of interest) be simulated with a fine level of resolution. We present a method of placing higher resolution adaptive mesh refinement grids when there is a small region of interest by using the adjoint equation.

GeoClaw and Adaptive Mesh Refinement

- Clawpack: finite volume methods to solve hyperbolic systems of PDEs [3].
- GeoClaw: extends this code base to solve geophysical flow problems [1].

When modeling tsunamis this code uses adaptive mesh refinement, refining cells if:

- the surface elevation is perturbed from sea level above a set tolerance,
- or if the speed of the water is above some set tolerance.

These approaches refine cells anywhere the tolerance is exceeded, even if the area of interest is only a subregion of the full solution domain.

The Adjoint Method

Suppose we are interested in calculating the value of a functional

$$J = \int_a^b \varphi(x)^T q(x, t_f) dx,$$

where the q is the solution to the time-dependent equation (**Forward Problem**):

$$q_t + A(x) q_x = 0$$

subject to some initial and boundary conditions. Note that for any time $t_r \in [t_0, t_f]$:

$$\int_a^b \int_{t_r}^{t_f} \varphi(x)^T \left(q_t + A(x) q_x\right) dt dx = 0. \tag{1}$$

Adjoint Problem: Solve

$$\hat{q}_t + (A(x)^T \hat{q})_x = 0$$

backwards in time from data $\hat{q}(x, t_f) = \varphi(x)$, with appropriate boundary conditions. Then integration by parts shows that (1) reduces to

$$\int_a^b \hat{q}^T(x, t_f) q(x, t_f)\, dx = \int_a^b \hat{q}^T(x, t_r) q(x, t_r)\, dx.$$

Hence J can be computed from the forward solution at an earlier time. More detail can be found in [4].

Implications of the Adjoint Method

- $\hat{q}(x, t_r)$ gives the *sensitivity* of J with respect to the data $q(x, t_r)$.
- If t_r is a regridding time, then the locations where the inner product $\hat{q}(x, t_r)^T q(x, t_r)$ is large at time t_r are the areas that will have a significant effect on the inner product at time $t = t_f$.
- We can apply adaptive mesh refinement to only these areas.
- The adjoint can also be applied to an estimate of the one-step error at t_r to refine based on an error tolerance.

The Shallow Water Equations

In two space dimensions the shallow water equations take the form

$$\begin{aligned} h_t + (hu)_x + (hv)_y &= 0 \\ (hu)_t + (hu^2 + \tfrac{1}{2} g h^2)_x + (huv)_y &= -ghB_x \\ (hv)_t + (huv)_x + (hv^2 + \tfrac{1}{2} g h^2)_y &= -ghB_y. \end{aligned}$$

Linearizing these equations about an ocean at rest lets us find the adjoint problem over the same domain as the forward problem, where the correct boundary conditions to use for the adjoint problem are zero normal velocity at all interfaces between any wet cell and dry cell and $\varphi(x)$ is chosen to highlight our region of interest.

Contact Information

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References

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[2] M. J. Berger and R. J. LeVeque. Adaptive mesh refinement using wave-propagation algorithms for hyperbolic systems. *SIAM J. Numer. Anal.*, 35(6):2298–2316, October 1998.

[3] Clawpack Development Team. Clawpack software, Accessed 2018. Version 5.3.

[4] B. N. Davis and R. J. LeVeque. Adjoint methods for guiding adaptive mesh refinement in tsunami modeling. *Pure Appl. Geophys.*, 173:4055–4074, 2016.

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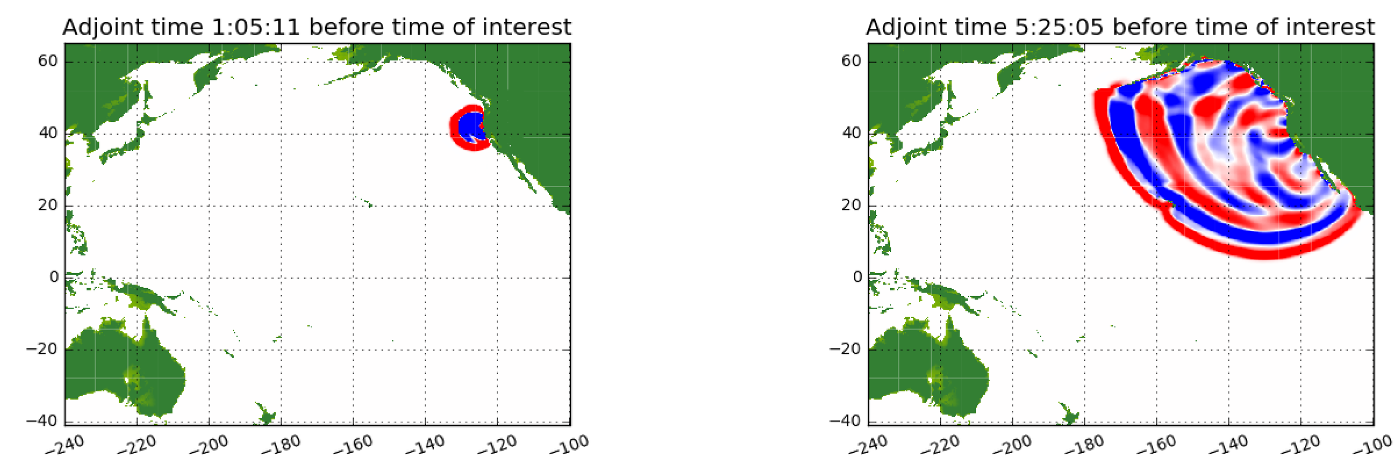
Adjoint Problem for Crescent City, California

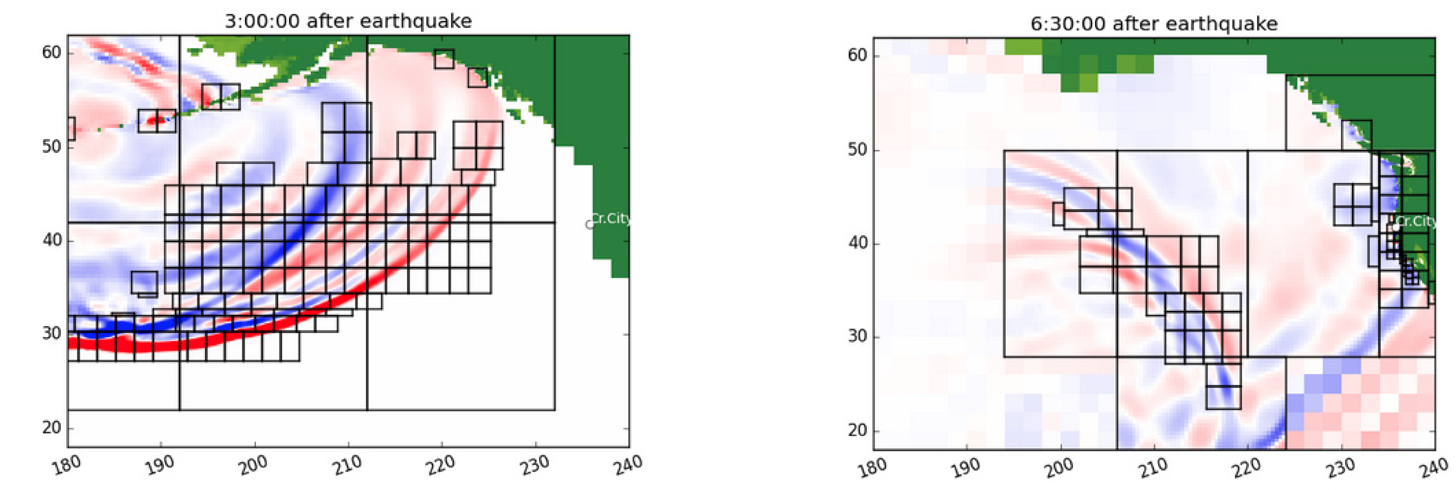
Suppose we are interested in the impact to Crescent City from tsunamis. Then $\varphi(x)$ is defined to be a Gaussian centered about the harbor in Crescent City, and spreads out from there.



Hypothetical Alaskan Tsunami

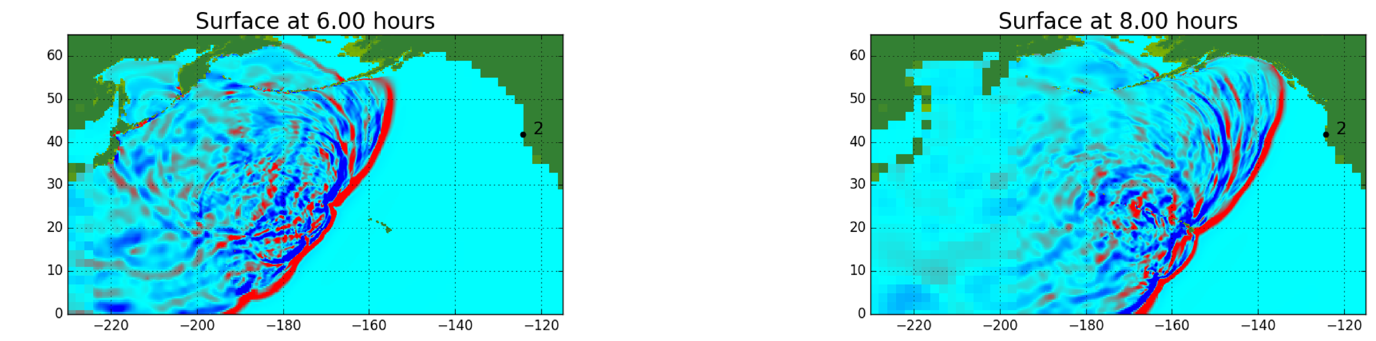
Suppose we are interested in the impact to Crescent City from a hypothetical tsunami originating in the Aleutian Islands, Alaska. The plots below show the surface of the ocean and the grid patches being used. Note that the adjoint method is:

- identifying a wave that reflects off of Hawaii and placing grid patches of greater refinement to capture that wave as well as the edge waves that are relevant, and
- ignoring the parts of the wave that will not reach Crescent City.

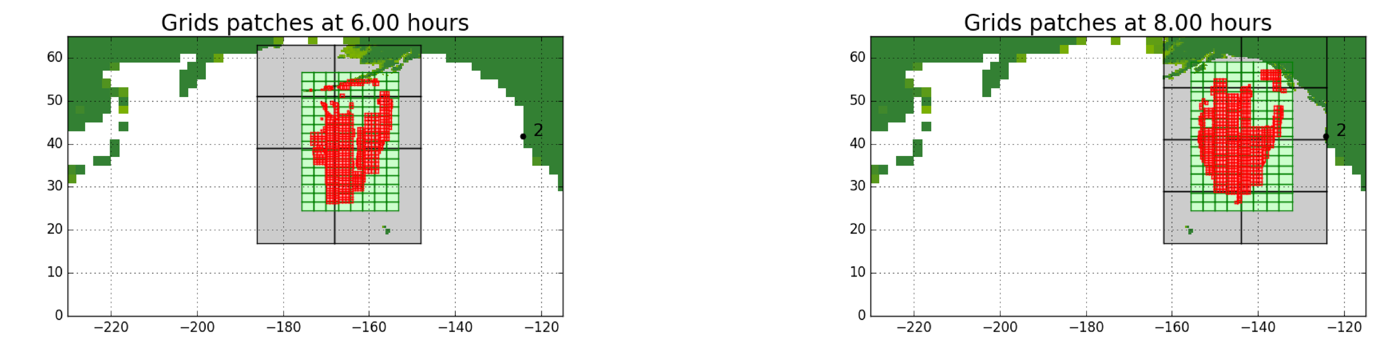


Japan 2011 Tsunami

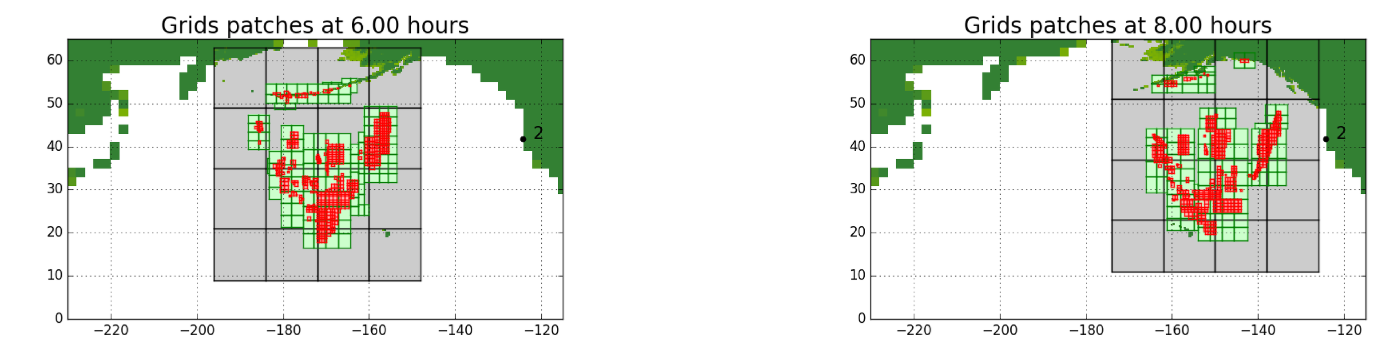
Now consider the tsunami originating from Japan in 2011, and suppose we are interested in the impact to Crescent City. For this tsunami, later waves were larger than the initially arriving waves. We can use the adjoint method to pick out waves that will arrive during a time range of interest that we specify.



Grid patches when focusing on waves arriving between 10.75 and 11.3 hours after tsunami:



Grid patches when focusing on waves arriving between 11.75 and 12.5 hours after tsunami:



Conclusion

Using the adjoint method to guide adaptive mesh refinement enables the targeted refinement of the regions of the domain that will influence a specific area of interest. [4] presents this work, and another paper is in preparation [5] to extend this work.

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