Abstract

This document details a plan to solve Group 1 (G.1)'s project prompt, together with a description of how such plan was derived and how the MATLAB program was made.

In the middle of February, 2015, the G.1 corporation came up with a project named Three Cannons Firework (T.C.F). The goal was to design a system of three cannons shooting firework bullet such that they collides at the exact same location on the air. This document was developed by a team of three creative and handsome G.R.C.C engineering students, code named Team 2 (T2).

Upon receiving the project details, T2 members spent the first two days individually researching, brainstorming, and developing ideas on how the task should be solved. Then the team spend one more day brainstorming together to arrive on a common solution. Another day was used to finish the MatLab program and the report altogether. Over the weekend, T2 also worked extra hours to create the animation to demonstrate their result, despite their busy schedule.

After putting in blood, sweat, and tears, T2 members are proud and excited to present their findings in hope that it will greatly assist G.1 corporation with their firework events, which they promised will be sure to improve human welfare, healthcare and the environment. T2 is privileged to cooperate with G.1 and the team looks forward to future collaboration.

Description

G.1 Corporation stated that they would like to to put on a fireworks show and they have three cannons. In order to get the greatest response out of the citizens, G.1 wanted all three projectiles to hit the same height at the same time. G.1 prompted T2 to determine the velocities at which to fire three different cannons at fixed angles and when to fire each relative to time zero (The time at which the middle cannon fired) so that all three collided.

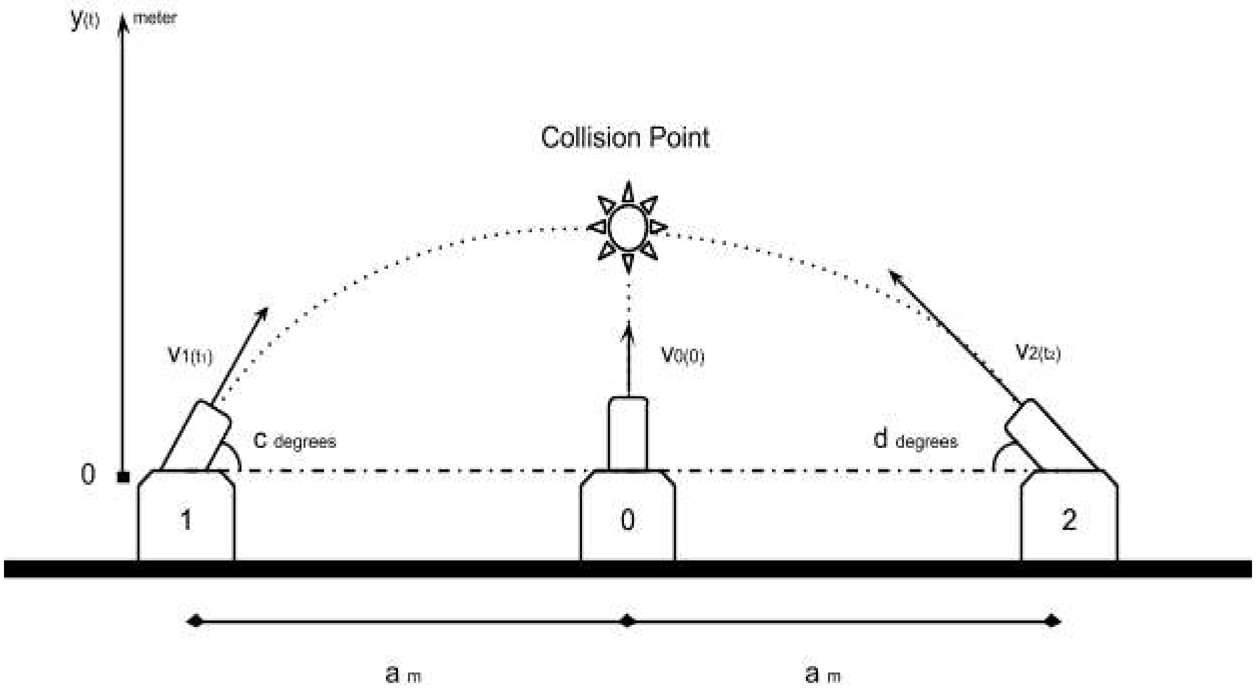

Figure 1: Three cannons Diagram

The angles of each cannon are fixed according to the diagram above, and the gravitational constant is 9.81 meters per second squared. Cannon number 0 shall fire when t=0. Cannon number 1 will be given according to the revised prompt given by Dr. Phillip.

Theory

T2 begins this section by making the assumption that frictions exerted by the atmosphere on the bullet are neglectable. Thus, T2 is able to classify this situation as an elementary kinetic motion problem. This problem first required the use of equating the proper kinematic equations used in physics to determine the flight path and time in air for projectiles. T2 was able to produce four equations in order to solve for the four unknowns. These values then allow us to determine the final time at which the collision occurs.

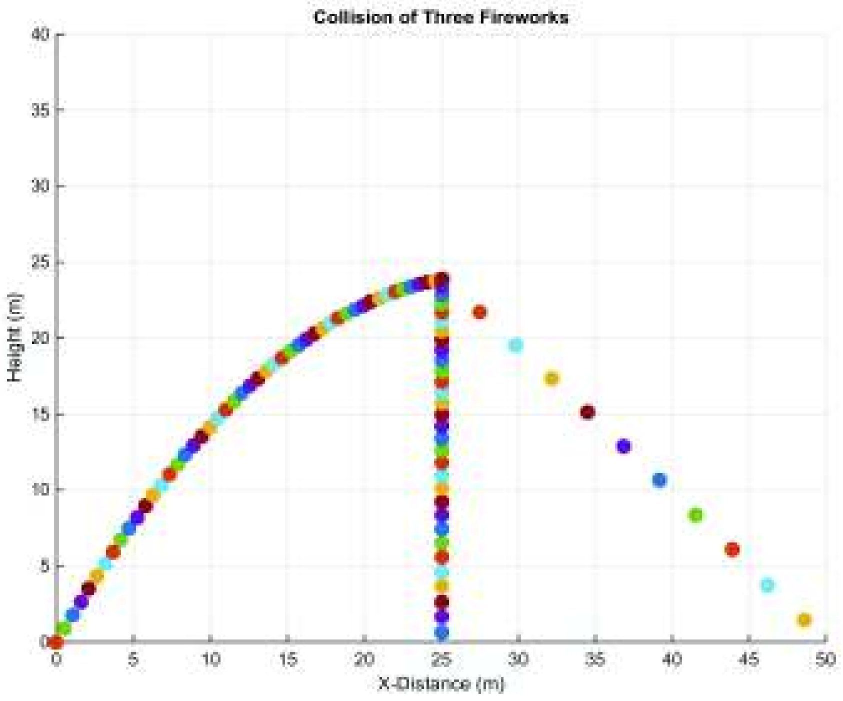

Figure 2: Three Projectiles at $t_{collision}$

All of the equations used can be seen below; this details T2's thought process of determining the appropriate physical equations and how those ideas were able to be turned into the proper functions. On the previous page, a graph has been included to represent how the code will produce G.1's answer visibly. When G.1 runs the code, they will be prompted for the

Velocity of the middle cannon, this will produce a short animation of all three projectiles being fired at their appropriate times and where they will collide.

Following the animation a window will appear that indicates all of the required variables so those watching the presentation are aware of their values.

The y_f is the same for all three cannons since the must collided at the same point, with the x_f at the middle as 25. The following is T2's system of equations:

- P_1 and P_2 meet at the same height
    - $y_1(t) = -½ *g * (t_1)^2 + v_1(t_1) * \sin(60°) + y_1(t_1)$
    - $y_2(t) = -½ *g * (t_2)^2 + v_2(t_2) * \sin(45°) + y_2(t_2)$
    - $y_0(t_{collision}) = y_1(t_{collision}) = y_2(t_{collision}) = y_f$

⇔ $F_1 = -½ *g * (t_1)^2 + v_1(t_1) * \sin(60°) + y_1(t_1) = -½ *g * (t_2)^2 + v_2(t_2) * \sin(45°) + y_2(t_2)$

- P_2 and P_0 meet at the same height
    - $y_2(t) = -½ *g * (t_2)^2 + v_2(t_2) * \sin(45°) + y_2(t_2)$
    - $y_0(t) = -½ * g * (t_0)^2 + v_0(t_0) + y_0(t_0)$

⇔ $F_2 = -½ *g * (t_2)^2 + v_2(t_2) * \sin(45°) + y_2(t_2) = -½ * g * (t_0)^2 + v_0(t_0) + y_0(t_0)$

- P_1 and P_2 meet at $x = 25$
    - $25 = 0 + v_1(t_1) * \cos(60°)$
    - $25 = 50 + v_2(t_2) * \cos(45°)$
    - $v_1(t_1) * \cos(60°) = 50 + v_2(t_2) * \cos(45°)$

⇔ $F_3 = v_1(t_1) * \cos(60°) - 50 - v_2(t_2) * \cos(45°)$

- Distance P_1 and P_2 travel: x-component of P_1 and P_2 were already set as equal so only one equation for P_1 and P_2 is needed.

- $25 = 50 + v_2(t_2) * \cos(45°)$

⇔ $F_4 = v_2(t_2) * \cos(45°) + 25$

Thus derived four equations with four unknowns:

With $x_1 = v_1$, $x_2 = v_2$, $x_3 = t_2$, $x_4 = t_0$, t_1 and v_0 as inputs.

- $F_1 = -½ *g * (t_1)^2 + x_1(t_1) * \sin(60°) + y_1(t_1) = -½ *g * (x_3)^2 + x_2(t_2) * \sin(45°) + y_2(x_3)$
- $F_2 = -½ *g * (t_2)^2 + x_2(x_3) * \sin(45°) + y_2(x_3) = -½ * g * (x_4)^2 + v_0(x_4) + y_0(x_4)$
- $F_3 = x_1(t_1) * \cos(60°) - 50 - x_2(t_2) * \cos(45°)$
- $F_4 = x_2(x_3) * \cos(45°) + 25$

$F = [F_1; F_2; F_3; F_4]$

The Jacobian matrix is thus:

$$J = [\begin{array}{llll} \sin(60°).*(t_1), & -\sin(45°).*(x_3), & -(x_2).*\sin(45°) + 9.81.*(x_3), & 0; \\ 0, & \sin(45°).*(x_3), & (x_2).*\sin(45°) - 9.81.*(x_3), & -v_0+ 9.81.*(x_4); \\ \cos(60°).*(t_1), & -\cos(45°).*(x_3), & -(x_2).*\cos(45°), & 0; \\ 0, & \cos(45°).*(x_3), & (x_2).*\cos(60°), & 0] \end{array}$$

The trajectory formed by the bullets have the shape of a parabola. We chose to use Newton's Method for solving nonlinear equations. Using a loop, the proper velocities and time can be determined by using:

x=Jacobian\-F

What this is doing is solving a system of equations and updating the x value which it will then re-enter back into the equation; each time this will narrow in on a more accurate answer than the previous iteration gave.

This concludes the Theory section.

Results

T2 thanks G.1 for choosing them to provide a solution to their problem. The code and any prompts which ask the user for an input have all been clearly marked and provide a guide to eliminate any confusion or questions. We are confident that we have provided you with a clear and easy to use a program that takes away any guess work and are sure that your problem has been solved accurately and to within a small margin of error. If you require further assistance with any other endeavors your company faces, we would be happy to work with you again.

Team Two will submits the following to G.1 corporation:

- This document which details each component of the project.
- A Matlab program with function as defined by G.1, which shall output an Excel file with information needed, and a picture of the graph.

We hope the event success.

Best Regards,

Team Two