A Numerical and Experimental Study of Mass Transfer in the Artificial Kidney

Introduction

Kidney disease is a major problem, affecting about 5% of the population and accounting for about 40,000 deaths each year in the United States. The total ESRD (End-Stage Renal Disease) population has approached 350,000 patients, requiring hemodialysis to live. The total costs including both Medicare and non-Medicare were $17.9 billion in 2000 [1].

In hemodialysis, an artificial kidney (also called as hemodialyzer, cylindrical in shape with about 2-5 cm in diameter and 15-25 cm in length, containing over 10,000 hollow fibers) is used to remove waste products (such as excess plasma water, uremic solutes, and toxins [2]) from blood. Dialysate (fresh dialysis solution) flows outside hollow fibers (shell-side flow), while blood goes inside hollow fibers (lumen-side flow) (Fig. 1a). Because the pressure and solute concentration between blood and dialysate sides are different, the waste products can be removed from blood to dialysate side through the wall of hollow fibers made of synthetic porous membrane. Artificial kidney is designed to mimic natural kidney containing millions of nephrons where the blood purification is taking place. Although artificial kidney is used very effectively to remove fluids and toxins, there are several functions of natural kidney it cannot achieve, such as regulating red blood cell production and blood pressure.

From an engineering perspective, an artificial kidney is actually a countercurrent mass exchanger. It is very important to study blood and dialysate flow in order to develop a more efficient artificial kidney. Previously, experiments were performed to optimize artificial kidney design and hemodialysis conditions by combining different kinds of membranes (such as cellulose triacetate, polysulfone, polyethersulfone, and polyparylethersulfone) and flow conditions (typical blood flow rates are from 200 to 500 ml/min, and dialysate flow rates from 500 to 800 ml/min). However, this experimental approach is very expensive and time consuming. In contrast, computer simulation may be an efficient and effective way to study the flows of blood and dialysate to predict optimal operating conditions and optimal design of artificial kidney for hemodialysis because it can save substantial time and reduce experimental cost.

A simple one-dimensional model has been used to simulate mass transfer in artificial kidney [3,4]. This model assumed that the solute concentration varied only in the axial direction, and could achieve good results by the help of some empirical correction. However, the solute concentration varies not only in the axial direction but also in the radial direction. The solute concentration in the bulk flow in a hollow fiber is different from that in the membrane surface because of solutes traversing through membrane or rejected by membrane (i.e., concentration polarization, the accumulation of molecules at the membrane surface). Jaffrin [5] reviewed various models of mass transfer, and he agreed that a rigorous calculation of the combined diffusion-convection mass transfer is very complex. Actually, the accuracy of the one-dimensional model by focusing on changes in one direction and ignoring changes in other directions is limited, and the rigorous calculation cannot be achieved by this model.

Gostoli et al. [6] developed an “equivalent annulus” model, which considers an equivalent single hollow fiber and assumes the blood flow within the fiber is laminar flow and dialysate flow outside the fiber is equivalent to the flow within a virtual annular region with symmetrical boundary. Gostoli applied this model with the assumption of uniform membrane wall mass flux to...
evaluate Sherwood numbers theoretically. The other assumption of this model is that the flow distribution around each hollow fiber is identical. However, recent MRI (magnetic resonance image) experiments [7,8] revealed that flow distribution in dialysate side is greatly nonuniform. Currently, high-flux membrane is widely used in the modern artificial kidney with high ultrafiltration (i.e., transmembrane flux from blood to dialysate side) and backfiltration (i.e., transmembrane flux from dialysate to blood side) rate, therefore the transmembrane flux is no longer uniform along the length of hollow fiber as assumed in Ref. [6]. Because there are several thousands of hollow fibers in the artificial kidney, the spacing between hollow fibers for dialysate flow is highly limited, and therefore the dialysate flow can be treated as the flow in the heterogeneous porous media [9,10], which is governed by Darcy's law. Lemanski and Lipscomb [9] presented a theoretical analysis of shell-side flows and their influence on mass transfer, and provided a framework for further investigation. However, their model assumed that there was no ultrafiltration and the solute concentration in lumen side was zero. Osuga et al. [10] used the same approach to predict the dialysate isobars, and numerical results showed that the dialysate isobars varied only in axial direction in the central part, which was supported by the MRI experiments. The other important issue is Darcy permeability, which is affected by many factors, such as fiber packing density, fiber size, packing geometry, and membrane material (roughness). Happel et al. [11] and Skartits et al. [12] reviewed the values of Darcy permeability for some specific material (carbon fiber) in different packing geometry (such as square and triangular array), but their results may not be applied in artificial kidney because the material and roughness in artificial kidney are different. However, we [13–15] recently designed a specific experiment system to measure Darcy permeability in the full-size artificial kidney.

This paper develops a modified porous media model with the consideration of both convection and diffusion by coupling lumen-side flow, shell-side flow, and transmembrane flow. Navier-Stokes equations are employed to simulate shell-side flow, and Kedem-Katchalsky equations are used to compute transmembrane flow. Experiments are performed to validate model predictions from numerical simulations. The detailed distribution of flow and solute concentrations are obtained and analyzed. The numerical results can serve as a guide for further design of optimal artificial kidney. The numerical and experimental studies are performed at water solution without using real blood. For small solutes, such as urea, the range of error between the clearance in water solution and real blood is small and of the order of 3%. For other solutes, particularly protein-bound solutes, the difference may be more substantial [2]. The model developed in this paper is applicable to small solutes, and numerical and experimental results from this study agreed well. The advanced model for protein-bound solutes is still under development.

**Modeling and Algorithm**

**Basic Assumption.** We made two major assumptions as follows:

1. Lumen-side flow is two-dimensional flow with only axial and radial components, and each hollow fiber is assumed to have the same flow rate in its inlet. The reason is that recent studies [7,8,16] showed that the lumen-side flow distribution in each hollow fiber is almost the same.

2. Shell-side flow is also a two-dimensional flow in the porous media with only axial and radial components. It is mainly because of special design of plate distributor in the dialysate inlet and outlet.
These two assumptions will be discussed in detail below. Because there exist ultrafiltration and backfiltration between lumen and shell side, the lumen-side flow may not be Hagen-Poiseuille flow any more (depending on the ultrafiltration and backfiltration rate) and therefore Navier-Stokes equations need to be solved in the appropriate boundary conditions. Figure 1(b) shows the computational domain of modeling, which focuses on the area of interest and actually represents an “ideal artificial kidney” with the assumption of same blood flow rate in the computational blood inlet of each hollow fiber and uniform flow distribution in the computational dialysate inlet and outlet.

Lumen-Side Flow. A schematic of computational domain of lumen-side flow is shown in Fig. 2. In clinical situation, the Reynolds number is around 1 therefore lumen-side flow is laminar. The mass transfer in the lumen side can be described by two-dimensional Navier-Stokes (N-S) equations in the steady state [17]:

Continuity equation:
\[ \frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0. \] (1)

Momentum equations:
\[ u \cdot \nabla u_r = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} \nabla^2 u_r , \]
\[ u \cdot \nabla u_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \nabla^2 u_z . \] (2)
(3)

Concentration equation:
\[ u \cdot \nabla C = D \nabla^2 C. \] (4)

Boundary condition:
Inlet: \[ Q = \frac{Q_{bin}}{N} \quad \text{or} \quad u_z(r) = \frac{2Q_{bin}}{N\pi R^3}(R^2 - r^2) ; \]
\[ u_r = 0 ; \quad C = C_{bin} . \] (5)

Outlet: \[ p = 0 . \] (6)

Membrane surface: \[ u_r = J_v ; \quad u_z = 0 ; \quad J_s = -D \frac{\partial C}{\partial r} + J_v C . \] (7)

Centerline: \[ u_r = 0 ; \quad \frac{\partial u_z}{\partial r} = 0 ; \quad \frac{\partial C}{\partial r} = 0 . \] (8)

where \( u = (u_r, u_z) \) is the velocity vector, \( u_r \) is the velocity component in the radial direction, \( u_z \) is the velocity component in the axial direction, \( C \) is the concentration of solute (such as urea and creatinine), \( p \) is the pressure, \( \rho \) is the density, \( \mu \) is the viscosity of solution, \( D \) is the diffusion coefficient of solute in the solution, \( Q_{bin} \) and \( C_{bin} \) are flow rate and solute concentration in the inlet, respectively. \( N \) denotes the number of hollow fibers. Each hollow fiber is assumed to have the same lumen-side flow rate at the computational blood inlet. \( R \) denotes the inner radius of hollow fiber. The velocity profile in the inlet is assumed as Hagen-Poiseuille flow distribution. \( J_s \) and \( J_v \) are solution and solute flux across membrane, which are calculated by Kedem-Katchalsky equations.

Transmembrane Flow. Kedem-Katchalsky (K-K) equations [18] are used to calculate mass transfer across the membrane if the distribution of pressure and concentration in lumen and shell sides are known:

\[ J_v = L_p(P_{bs} - P_{ds}) - \sigma L_p R T (C_{bs} - C_{ds}) , \]
\[ J_s = C^*_v (1 - \sigma) J_v + P_s (C_{bs} - C_{ds}) , \] (9)
(10)

where \( J_v \) (m³/m²/s) is solution flux across the membrane, \( J_s \), (kg/m²/s) is solute flux across a membrane, \( L_p \) (m/s/Pa) is hydraulic permeability of a membrane, \( P_s \) (m/s) is solute diffusive permeability of a membrane, \( \sigma \) is the reflection coefficient, \( C^*_v \) (kg/m³) is the average of solute concentration inside membrane, \( C_{bs} \) (kg/m³) and \( P_{bs} \) (Pa) are solute concentration and hydrostatic pressure in the lumen-side/blood-side membrane surface, respectively, \( C_{ds} \) (kg/m³) and \( P_{ds} \) (Pa) are the solute concentration and hydrostatic pressure in the shell-side/dialysate-side membrane surface, respectively. The schematic is shown in Fig. 3. K-K equations indicate that the solute mass transfer across the membrane is determined by both convection and diffusion (the first part in the right of Eq. (10) is the contribution of convection, and \( J_s \) has the unit of m³/s; the second part is the contribution of diffusion determined by concentration difference). Because the pressure and solute concentration change along the length of hollow fiber, \( J_v \) and \( J_s \) will also change.

Hydraulic permeability \( (L_p) \), diffusive permeability \( (P_s) \), and reflection coefficient \( (\sigma) \) are intrinsic parameters, which only depends on the membrane property regardless of different flow conditions. Hydraulic permeability describes the volume of solution passing through unit membrane area, per unit pressure and per unit time. Diffusive permeability describes the amount of solute passing through unit area of membrane, per unit time and per unit concentration difference across the membrane. Reflection coefficient means the fraction of solute that cannot pass through the membrane under certain specified conditions of flow flow., \( L_p \), \( P_s \), and \( \sigma \) will be determined by the experiments.

Shell-Side Flow. There is a raised collar in both inlet and outlet of shell-side flow. Figure 4 shows the geometry configuration near the dialysate inlet. No hollow fiber is contained in the raised collar, so the collar has lower resistance than the fiber bundle. A small plate distributor in the raised collar is facing the inlet, and its function is to prevent the dialysate from going into the fiber bundle directly from inlet. When the dialysate comes into the inlet, it will first impinge on the distributor, then go around the
collar, finally penetrate into the fiber bundle [16]. Also, another small plate distributor is facing the outlet. In the same way, when the dialysate goes out to the outlet, it will first go out from the fiber bundle circumferentially, then go around the collar, finally concentrate to the outlet and go out.

Since the distributors are functioned as keeping the dialysate come into the fiber bundle uniformly after the inlet, and go out uniformly from fiber bundle before concentrating to the outlet, one assumption we made here is to treat the dialysate flow as two-dimensional flow only with radial flow \( u_r \) and axial flow \( u_z \) in a porous media (Fig. 5). The computational domain is the area just containing hollow fibers, and does not include the raised collar and dialysate inlet and outlet.

Darcy equations are employed to simulate dialysate flow (Eqs. (11)–(14)).

Continuity equations:

\[
\frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} = S_m. \tag{11}
\]

Momentum equations:

\[
u_r = -\frac{1}{\mu} k_{rr} \frac{\partial p}{\partial r}, \tag{12}
\]

\[
u_z = -\frac{1}{\mu} k_{zz} \frac{\partial p}{\partial z}. \tag{13}
\]

Concentration equation:

\[
u_r \frac{\partial (rC)}{\partial r} + \nu_z \frac{\partial C}{\partial z} = S_s. \tag{14}
\]

Boundary condition:

Inlet: \( Q = Q_{din} \) or \( u_r = -\frac{Q_{din}}{2\pi R_d L_r}, u_z = 0 \). \tag{15}

Outlet: \( p = 0 \). \tag{16}

Shell side and centerline: \( u_r = 0 \). \tag{17}

Tube sheet: \( u_z = 0 \). \tag{18}

where \( k_{rr} \) and \( k_{zz} \) are Darcy permeability in the radial and axial direction, respectively, and determined by experiments, \( S_m \) is the solution source term which means how much solution comes from blood to dialysate side per unit volume, \( S_s \) is the solute source term which means how much solute comes from blood to dialysate side per unit volume, \( Q_{din} \) is the flow rate in the dialysate inlet, \( R_d \) is the shell radius, \( L_r \) is the width of the raised collar. Velocity profile in the dialysate computational inlet is assumed as uniform.

The solution source term \( S_m \) and solute source term \( S_s \), which come from the mass exchange between blood flow and dialysate flow, are very important. When Lemanski and Lipscomb [9] applied porous media model for dialysate flow, they assumed \( S_m = S_s = 0 \). The contributions of \( S_m \) and \( S_s \) can be neglected when transmembrane fluxes are low, but this assumption breaks down under certain operating conditions when high ultrafiltration and backfiltration occur. Now we propose a new method to calculate \( S_m \) and \( S_s \).

Consider a fluid control volume (Fig. 6), whose cross section area is \( A \) and length is \( \Delta z \). We can estimate the number of hollow fibers \( N_H \) in this control volume by:

\[
N_H = \frac{A}{A_H}. \tag{19}
\]
where \( A_H \) is cross section area of one hollow fiber, \( f \) is packing density which is how much percentage of space occupied by hollow fibers. Then membrane area \( A_m \) in this control volume can be determined by:

\[
A_m = N_H \pi d_H \Delta z,
\]

(20)

where \( d_H \) is the diameter of hollow fiber.

So we can calculate \( S_m \) by:

\[
S_m = \frac{J_v A_m}{A \Delta z},
\]

(21)

where \( J_v \) is solution flux across membrane, which can be calculated from K-K equations (Eq. (9)).

In the same way, \( S_i \) can be calculated by:

\[
S_i = \frac{J_i A_m}{A \Delta z},
\]

(22)

where \( J_i \) is solute flux across the membrane, which can be calculated from K-K equations (Eq. (10)).

Solution Methodology. Shell-side flow area is divided into a number of small computational meshes. Each mesh still includes many hollow fibers (Fig. 7). In each computational mesh, dialysate solute concentration and pressure are unique and applied to all the parts of hollow fibers contained in this mesh. The process of computing is showed in Fig. 8. In the discretization of N-S and Darcy equations, the staggered grid system [19] is used. The pressure and concentration components are stored in the center of control volume, while the velocity components are stored in the edges of control volume. The SIMPLER algorithm developed by Patankar [19] is used to calculate the pressure, velocity, and concentration.

Several typical numerical examples were used to verify the programming code, such as laminar flow in the tube, fluid flow in sudden expansion [20], and driven cavity flow [21]. The mesh refinement study was performed to make sure that the results were not affected (the relative error below 0.1%) by numerical approximation error.

Experimental Methods

Clearance (K) is an important measure of the performance of artificial kidney. The physical meaning of \( K \) is the equivalent volume flow rate at which the toxic solutes are cleaned completely from the blood, which can be easily used to calculate the dialysis time for patients. \( K \) is calculated from the following equation:

\[
K = \frac{Q_{\text{bin}} C_{\text{bin}} - Q_{\text{bout}} C_{\text{bout}}}{C_{\text{bin}}},
\]

(23)

where \( Q_{\text{bin}} \) and \( Q_{\text{bout}} \) are blood (lumen-side) flow rates in blood (lumen-side) inlet and outlet, respectively; \( C_{\text{bin}} \) and \( C_{\text{bout}} \) are solute concentration in blood (lumen-side) inlet and outlet, respectively.

Experiment setup to measure clearance (K) is shown in Fig. 9. Blood reservoir contained deionized water with test solutes, such as urea, while the dialysate reservoir contained deionized-water based dialysate. Other experimental instruments include the arterial blood tubing (Baxter Arterial blood Tubing Set #5M4283M), venous blood tubing (Baxter Arterial blood Tubing Set #5M4484M), two balances (Model No.:EA15DCE-I, Sartorius AG, Goettingen, German), two heaters (Cimarec™ stirring Hot Plates, Model No.:SP47235, BarnsteadThermolyne Co., Dubuque, IA, USA), and two roller pumps (Drive, Model No.:7521-40; Header, Model No.:7518-12; Tubing, Model No.:96410-18; Cole-Parmer Instrument Co., Vernon Hills, IL). One pump was used to push blood flow through the blood compartment, and the other pump was used to push dialysate flow through the dialysate compartment. The blood/dialysate flow rate was determined by measuring the mass increase of blood/dialysate in reservoir on top of
balance in unit time. Samples were collected from the inlet and outlet of blood and dialysate, and assayed on the Cobas Mira-S® autoanalyzer (Roche Diagnostic Systems Inc., Somerville, NJ).

Previous studies [7,8,22] have shown nonuniform flow distribution in the dialysate flow. A new experimental method was introduced here to evaluate this effect (local clearance of artificial kidney) and validate numerical results. In this experiment, three concentric annular rings, each with same area, are made (see Fig. 10). It is assumed that hollow fibers are uniformly distributed therefore each ring has the same number of hollow fibers. Experiments to measure clearance are performed at three cases, each with one ring open (the lumen-side flow can go inside hollow fibers in the open ring area) while the other two rings are sealed (the lumen-side flow cannot go inside hollow fibers in the sealed ring area). Thus, the local clearance in each ring can be obtained [23].

The artificial kidney we chose in our study is CT190G (Baxter Healthcare Co., McGaw Park, IL, USA; cellulose triacetate membrane; length: 20.32 cm; inner diameter of hollow fiber: 200 μm; membrane thickness: 15 μm; inner diameter of shell: 3.50 cm; number of hollow fibers: 12,000). The test solute is urea (60 Dalton; diffusion coefficient $D = 1.81 \times 10^{-9} \text{ m}^2/\text{s}$), which represents the small molecular weight solutes of vital interest in end-stage renal-disease (ESRD) patients, and is the solute most frequently monitored in clinical practice. Different solutes have different values of $P_s$ and $\sigma$ for one hollow-fiber membrane, but $L_p$ only depends on the type of membrane. The urea transport properties of CT190G are measured as follows [13,15]:

$$L_p = 1.15 \times 10^{-10} \text{ m/s/Pa}, \quad P_s = 9.76 \times 10^{-6} \text{ m/s}, \quad \sigma = 0.$$  

The Darcy permeabilities in CT190G are [14,15]: $k_{rr} = 4.16 \times 10^{-11} \text{ m}^2$, $k_{zz} = 1.29 \times 10^{-9} \text{ m}^2$.

### Results and Discussions

Table 1 compared experimental and numerical results of clearance of urea in different lumen-side flow ($Q_b = 300, 400 \text{ ml/min}$) and shell-side flow ($Q_d = 200, 300, 400, \text{ and } 500 \text{ ml/min}$). The

<table>
<thead>
<tr>
<th>$Q_b$ (ml/min)</th>
<th>$Q_d$ (ml/min)</th>
<th>Expt.*</th>
<th>Num.</th>
<th>Error</th>
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<tr>
<td>300</td>
<td>200</td>
<td>196 ± 4.0</td>
<td>189</td>
<td>3.6%</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>235 ± 7.4</td>
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<td>300</td>
<td>400</td>
<td>267 ± 6.2</td>
<td>241</td>
<td>9.7%</td>
</tr>
<tr>
<td>400</td>
<td>200</td>
<td>212 ± 9.8</td>
<td>210</td>
<td>1.0%</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>294 ± 5.0</td>
<td>276</td>
<td>6.1%</td>
</tr>
<tr>
<td>400</td>
<td>500</td>
<td>306 ± 8.2</td>
<td>295</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

*Experimental data are shown in MEAN±STD, sample size $n = 3$.

![Fig. 10 Schematic of concentric annual rings in shell side. Each ring has the same area. Each experiment was performed on the dialyzer with one ring open and other two rings sealed.](image)

![Fig. 9 Clearance measurement: experimental setup](image)

![Fig. 11 Numerical and experimental results of local urea clearance in different annular ring with different dialysate flow rate. “Inner” in the graph denotes the inner ring open only with two other rings sealed, “middle” denotes the middle ring open only, and “outer” denotes the outer ring open only. “exp.” in the graph means experimental result, and “num.” means numerical result. Error bar indicates MEAN±STD, sample size $n = 3$.](image)
urea concentration in the lumen inlet is set as 70 mg/dL. The numerical results are in good agreement with experimental results within 10% error, compared to 25% error if the solution source term $S_a$ is neglected in the simulation. When $Q_b$ is fixed, urea clearance increases with the increasing $Q_d$. When $Q_d$ is fixed, urea clearance also increases with the increasing $Q_b$.

Figure 11 compared numerical results with experimental results in local urea clearance at different annular rings in different dialysate rates ($Q_d = 500, 800$, and $1000$ ml/min). The lumen-side flow rate and urea concentration in the inlet are set as $120$ ml/min and $100$ mg/dL, respectively. Numerical results agreed well with experimental results within 6% error. The clearance in the outer ring is higher than that in the inner ring when $Q_d = 500$ and $800$ ml/min (9.6% and 8.6% higher in experimental and numerical results respectively when $Q_d = 500$ ml/min; 5.8% and 4.6% higher in experimental and numerical results, respectively, when $Q_d = 800$ ml/min), while the clearance in the outer ring is almost the same as that in the inner ring when $Q_d = 1000$ ml/min (1.9% and 1.7% higher in experimental and numerical results, respectively).

These results show that nonuniformity of dialysate flow distribution is significant in low ($Q_d = 500$ ml/min) or moderate ($Q_d = 800$ ml/min) flow rate, but much less significant in high ($Q_d = 1000$ ml/min) flow rate.

It is very important and necessary to study the distribution of velocity, pressure, and concentration in detail, which can give us a deep insight into the flow pattern in the artificial kidney. Figures 12–15 show some numerical results for CT190G with $Q_b = 360$ ml/min, $Q_d = 500$ ml/min, $C_b = 48.0$ mg/dL. The solution only contains urea. Figure 12 shows the velocity distribution in the dialysate side. The axial flow is dominant in the middle range of dialysate side, but the radial flow is significant in the location near inlet and outlet. Figure 13 shows the distribution of pressure. The pressure is uniform in the middle range, but nonuniform near dialysate inlet and outlet, which are consistent with Osuga's findings [10]. Figure 14 shows the distribution of solute concentration, which indicates that the concentration is not uniform in each cross section where the concentration in the center is larger than that in the periphery. The highest concentration is not in the dialysate outlet, but in the centerline below the outlet. To the best of our knowledge, it is the first time in the literature to get the distribution of concentration numerically. Figure 15 shows dialysate axial velocity distribution in three different cross sections along the length of hollow fiber. The velocity distribution is nonuniform in each cross section. The axial velocity in the periphery was about 30% higher than that in the centerline in sections (1) (near dialysate-outlet section) and (3) (near dialysate-inlet section), while it was about 5% higher in section (2) (middle section). The main reason is the inlet/outlet effect and different Darcy permeability in different direction (i.e., there is lower resistance for axial flow than radial flow). The nonuniformity of flow distribution contributes to the nonuniform concentration distribution in the dialysate side since the solute transport across the membrane is determined by flow and concentration distribution in both blood and dialysate sides (K-K equations).

From the designer's perspective, the more uniform the distribution of dialysate velocity and concentration, the higher perfor-

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**Fig. 12** Velocity field in shell side of CT190G (the axial length is 24.0 cm and the radius is 1.7 cm). Dialysate flow rate $Q_d = 500$ ml/min. The radial velocity dominates in the dialysate inlet and outlet, while axial velocity prevails in the middle range. The velocity vector depicts both magnitude and direction of velocity at the location of the starting point of the arrow.

**Fig. 13** Distribution of pressure in shell side of CT190G (the axial length is 24.0 cm and the radius is 1.7 cm). Dialysate flow rate $Q_d = 500$ ml/min. The pressure in the dialysate outlet is set to zero.

**Fig. 14** Distribution of concentration in shell side of CT190G (the axial length is 24.0 cm and the radius is 1.7 cm). Dialysate flow rate $Q_d = 500$ ml/min. The urea concentration in the dialysate inlet is set to zero.
formance of artificial kidney. To achieve this goal, the structure of artificial kidney may be modified by changing (1) housing/shell shape and dimension; (2) the dimension and shape of two circumferential headers in dialysate inlet and outlet; (3) position and number of dialysate entry openings. This porous media model can be used to evaluate these modifications and help design more optimal artificial kidney to achieve high clearance of uremic solutes/toxins.

Conclusions

Lumen-side flow (governed by the N-S equations), shell-side flow (governed by the Darcy equations), and transmembrane flow (governed by the K-K equations) were coupled to simulate the mass transport in artificial kidney. Experimental results agreed well with numerical results within 10% error. Numerical results show the nonuniform distribution of velocity and solute concentration in shell side due to the inlet/outlet effect and Darcy permeability. In the section near to the inlet and outlet, the axial velocity in the periphery is much higher than that in the centerline. The nonuniformity of velocity distribution is a main reason to cause the nonuniformity of concentration distribution. The numerical model developed here may be used to help design more efficient artificial kidney and achieve the optimal operating conditions.

Acknowledgment

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Nomenclature

\[ C = \text{solute concentration} \]
\[ D = \text{free diffusion coefficient of solute} \]
\[ J_s = \text{solute flux across membrane} \]
\[ J_u = \text{solution flux across membrane} \]
\[ K = \text{clearance} \]
\[ K-K = \text{Kedem-Katchalsky} \]
\[ k_{rr} = \text{Darcy permeability in radial direction} \]
\[ k_{zz} = \text{Darcy permeability in axial direction} \]
\[ L_p = \text{hydraulic permeability} \]
\[ N-S = \text{Navier-Stokes} \]
\[ p = \text{pressure} \]
\[ P_s = \text{diffusive permeability} \]
\[ Q_{bin} = \text{lumen-side (blood) flow rate in the inlet} \]
\[ Q_{lin} = \text{shell-side (dialysate) flow rate in the inlet} \]
\[ Q_{bo} = \text{lumen-side (blood) flow rate in the outlet} \]
\[ C_{bin} = \text{solute concentration in the lumen-side (blood) inlet} \]
\[ C_{bo} = \text{solute concentration in the lumen-side (blood) outlet} \]
\[ u_r = \text{velocity in radial direction} \]
\[ u_z = \text{velocity in axial direction} \]
\[ \sigma = \text{reflection coefficient} \]
\[ \rho = \text{density} \]
\[ \mu = \text{viscosity} \]

References


