Lookback Options under the CEV Process: A correction

Phelim P. Boyle
Centre for Advanced Studies in Finance
University of Waterloo
Waterloo, Ontario
Canada N2L 3G1

and

Yisong "Sam" Tian
Department of Finance
to be changed to York address

and

Junichi Imai
Centre for Advanced Studies in Finance
University of Waterloo
Waterloo, Ontario
Canada N2L 3G1

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Phelim P. Boyle,Yisong S. Tian and Junichi Imai*

Abstract

Boyle and Tian(1999) developed a trinomial lattice method for valuing lookback options and barrier options under the CEV process. In the case of lookback options it turns out that this method produces values which are not quite accurate. In this note we discuss the source of the error and provide corrected numerical values. These revised values were obtained using the recently developed method of Davydov and Linetsky(1999). We also confirmed their results using Monte Carlo simulation. For both standard options and barrier options the Boyle-Tian lattice approach gives correct numerical values under the CEV assumption.

I. Introduction

Boyle and Tian (1999) proposed a trinomial lattice method to value certain types of exotic options. under the CEV process. In particular they used this approach to value both barrier and lookback options. Subsequently Linetsky and Davydov(1999) developed a closed form approach that is applicable to the CEV process. Linetsky and Davydov found that their results agreed with those of Boyle and Tian for barrier options under the CEV process but that there were systematic differences in the case of lookback options.

In this note we explain why the two sets of results differ in the case of lookback options under the CEV process. We confirm that the results of of Linetsky and Davydov are correct. We provide corrected numbers for the lookback examples in the Boyle-Tian paper. In addition we describe a simple Monte Carlo method that can be used to value the lookback options under the CEV process. We illustrate that the Monte Carlo method reproduces the Davydov-Linetsky results.

*We would like to thank Vadim Linetsky who discovered the mistake in our original paper and pointed it out to us. We are also grateful to Dmitry Davydov and Vadim Linetsky for providing us with numerical values for lookback option sunder their method.

The outline of the remainder of this note is as follows. In the next section we recall some features of the CEV process and describe the trinomial model of Boyle and Tian. then we discuss the origin of the error in the original paper in valuing lookbacks. We present the accurate prices from the Davydov-Linetsky method.

1 The Trinomial lattice method for the CEV process

Under the CEV process the stock price $\{S_t, t \geq 0\}$ follows the following diffusion process under the risk-neutral measure Q,

$$dS_t = rS_t dt + \sigma S_t^{\frac{\alpha}{2}} dB_t \tag{1}$$

where r, σ and α are constants. and $\{B_t, t \geq 0\}$ is a standard Weiner process. If $\alpha = 2$ we are back to the standard lognormal diffusion case.

Because the diffusion term is non-constant it is not possible to construct a standard lattice in the usual way. To surmount this problem Boyle and Tian transform this process to a Bessel process, y, with a constant diffusion term. The advantage of this transformation is that it is easy to construct a trinomial tree based on the y process. Once the tree is constructed in y-space we can then transform back to the original variable and implement the valuation. The details are given in the Boyle-Tian paper under the CEV process. The method works well for standard options and for barrier options. under the CEV process.

In the case of floating strike lookback options the option payoff depends on two stochastic variables: the terminal asset price and the realized extremum over the path. Boyle and Tian adjusted a method developed by Babbs(1992) for the lognormal case to handle this situation. There are two steps in the Babbs procedure. First the asset price itself is used as the numeraire. The second is to introduce a reflecting barrier at the origin. The Babbs procedure works very well in the case of the lognormal model when $\alpha = 2$. In this case the grid formed using the log of the asset price has constant volatility and when we reflect at the origin the reflected points end up lying on the original grid. This prevents the occurrence of a bushy tree after reflection occurs.

Boyle and Tian explain their approach in section V of their paper. First they construct a tree with constant volatility in the y space. Then they transform back W-space where $W_t = log(S_t)$. The lookback call can be valued by constructing a lattice based on W but with a reflecting barrier. The problem is that after reflection the grid points will not lie on the original grid as they do in the lognormal case. This means that reflected portion of the tree becomes bushy and the attempt to remove the path dependence is foiled. This point was not picked up in the Boyle Tian paper and it means that the numbers in Tables 4 and 5 of their paper for lookback options are not accurate.

2 Accurate prices for lookbacks under the CEV process

Davydov and Linetsky(1999) have developed a unified framework for valuation of a variety of claims where the underlying asset follows a general one-dimensional diffusion. The CEV case represents a special case of their model and they obtain a variety of closed form expressions for the value of different exotic options under the CEV process. In particular they obtain exact expressions for the prices of floating strike lookback calls and lookback puts. In Table One we reproduce their results for the lookback options considered in Table 5 of Boyle and Tian. We also display the original Boyle-Tian results for completeness. We note that the Boyle-Tian results are correct for the $\alpha = 2$ case but that there are systematic differences as we move away from $\alpha = 2$. The largest bias for these parameter values is less than 5%. but we should stress that the deviation could be much larger for other parameters.

3 A Monte Carlo Approach to pricing Lookbacks under the CEV process

We have developed a simple but accurate approach to pricing lookbacks under the CEV process. The standard Monte Carlo method does a poor job in pricing continuously monitored lookbacks even under the lognormal assumption. This is because when we discretize the stochastic differential equation and simulate it at discrete points we lose information about the part of the path between observation dates. Andersen and Brotherton-Ratcliffe(1998) discuss this point and provide numerical examples to illustrate the bias. For a one year lookback put with plausible parameter vales and a stock price of 100 the bias is around 5%. Andersen and Brotherton-Ratcliffe demonstrate how to correct for this bias.

We can use their procedure to value lookback options under the CEV process In addition we can reduce the variance further by noting that for the lognormal case $\alpha = 2$ we have a very simple closed form expression for the lookback option prices. Hence the case $\alpha = 2$ forms a very natural control variate for the problem. In table Two we provide the simulated option values incorporated both the Andersen-Brotherton-Ratcliffe modification and the control variate. We note that the resulting values are extremely accurate and that they are consistent with the accurate results obtained by Davydov and Linetsky.

Table One

Lookback Option Prices for the CEV Process with Different α Values: Comparison of corrected values with those in Boyle Tian paper

This table reports corrected prices for lookback call and put options. The accurate values were computed used the Davydov-Linetsky procedure. The lookback period is the full term to maturity, and the option contract was initiated prior to today. The current price of the underlying asset is 100, time to maturity is 6 months, the risk-free rate is 0.10 per annum, and the instantaneous volatility (of the percentage change in stock price) is 0.25 per annum. equal to the current minimum ($S^0_{\min}$) or maximum ($S^0_{\max}$) value of the corresponding lookback option.

I. Lookback Call options						
$S^0_{\min}$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 1.5$	$\alpha = 2$	max. %difference
	Lookback call prices: accurate values(Davydov and Linetsky)					
90	18.835	18.654	18.486	18.330	18.182	
95	16.844	16.719	16.567	16.427	16.296	
100	16.169	16.017	15.879	15.753	15.636	
	Lookback call prices (Boyle and Tian)					
90	19.018	18.764	18.545	18.351	18.182	0.97
95	17.215	16.922	16.680	16.475	16.297	2.20
100	16.811	16.418	16.104	15.846	15.633	3.97

II. Lookback Put options						
X or $S^0_{\max}$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 1.5$	$\alpha = 2$	max. %difference
	Lookback put prices: accurate values					
100	11.262	11.488	11.731	11.995	12.283	
105	11.654	11.887	12.138	12.409	12.704	
110	12.858	13.105	13.369	13.653	13.961	
	Lookback put prices (Boyle and Tian)					
100	11.778	11.864	11.974	12.113	12.284	4.58
105	11.973	12.123	12.293	12.485	12.705	2.74
110	13.049	13.247	13.463	13.700	13.961	1.49

Table Two
Lookback Option Prices for the CEV Process with Different α Values: Monte Carlo estimates

This table reports Monte Carlo estimates for the prices of lookback call and put options. The se estimates incorporate both the Andersen Ratcliffe adjustment and the control variate. The lookback period is the full term to maturity, and the option contract was initiated prior to today. The current price of the underlying asset is 100, time to maturity is 6 months, the risk-free rate is 0.10 per annum, and the instantaneous volatility (of the percentage change in stock price) is 0.25 per annum. equal to the current minimum ($S^0_{\min}$) or maximum ($S^0_{\max}$) value of the corresponding lookback option.

I. Lookback Call options					
$S^0_{\min}$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 1.5$	$\alpha = 2$
	Lookback call prices : standard errors in brackets				
90	18.837	18.655	18.487	18.330	8.182
90	(0.002)	(0.002)	(0.001)	(0.001	(0.000)
95	16.886	16.720	16.569	16.426	16.296
95	(0.002)	(0.002)	(0.001)	(0.001)	(0.00)
100	16.170	16.018	15.880	15.753	15.636
100	(0.002)	(0.002)	(0.001)	(0.001)	(0.000)
	Accurate Lookback call prices (Davydov and Linetsky)				
90	18.835	18.654	18.486	18.330	18.182
95	16.884	16.719	16.567	16.427	16.296
100	16.169	16.017	15.879	15.753	15.636

II. Lookback Put options						
X or $S^0_{\max}$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 1.5$	$\alpha = 2$	max. %difference
	Lookback put prices: standard errors in brackets					
100	11.262	11.488	11.732	11.995	12.283	
100	(0.002)	(0.001)	(0.001)	(0.001)	(0.000)	
105	11.654	11.887	12.138	12.409	12.704	
105	(0.002)	(0.001)	(0.001)	(0.001)	(0.000)	
110	12.858	13.105	13.369	13.653	13.961	
110	(0.002)	(0.001)	(0.001)	(0.001)	(0.000)	
	Lookback put prices: accurate values)					
100	11.262	11.488	11.731	11.995	12.283	
105	11.654	11.887	12.138	12.409	12.704	
110	12.858	13.105	13.369	13.653	13.961	

References

Andersen Leif and R Brotherton-Ratcliffe. “Exact Exotics.” Chapter 11, MONTE CARLO, Methodologies and Applications for Pricing and Risk Management, Risk Books, 1998.

Babbs, S. “Binomial Valuation of Lookback Options.” Working paper, *Midland Global Markets* (1992).

Boyle, Phelim, P., and Sam (Yisong) Tian. “ Pricing Lookback and Barrier Options under the CEV Process, ” *Journal of Financial and Quantitative Analysis* 34,2, (1999), 241-264.

Davydov Dmitry and Vadim Linetsky. “ The Valuation and Hedging of Barrier and Lookback Options for Alternative Stochastic processes. ” Working Paper(1999), Financial Engineering Program. University of Michigan, 611 Tappan Street, Ann Arbor, MI, 48109.