American-style Puts under the JDCEV Model: A Correction

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Abstract

Nunes (2009) prices American-style standard call and put options under the geometric Brownian motion, CEV, and JDCEV models, using an optimal stopping approach that is based on the first passage time density of the underlying asset price through the early exercise boundary. Under the JDCEV model, the solution provided by Nunes (2009, Equation 56) for the recovery component of the American-style put is wrong. This note corrects Nunes (2009, Equation 56), clarifies how the first hitting time density shall be determined, and compares the numerical results obtained with the static hedge approach recently offered by Ruas et al. (2013) for the JDCEV model.
I. Introduction

Nunes (2009) proposes an optimal stopping approach for the valuation of American-style options that is valid for any Markovian and diffusion underlying price process—such as the geometric Brownian motion or the CEV process—as well as for any parameterization of the (unknown) exercise boundary. The proposed approach only requires that the underlying price process provides a viable valuation method for European-style options and for its transition density function.

Nunes (2009, Section VII) extends this approach to the JDCEV framework of Carr and Linetsky (2006), where the price $S$ of the underlying defaultable stock is modeled, under the equivalent martingale measure $Q$, through the stochastic differential equation

$$
\frac{dS_t}{S_t} = [r_t - q_t + \lambda(t, S)] dt + \sigma(t, S) dW_t^Q,
$$

with $S_{t_0} > 0$, and where the (short-term and risk-free) interest rate $r_t$ and the dividend yield $q_t$ are deterministic functions of time, $\sigma(t, S)$ represents the instantaneous volatility of equity returns, $\lambda(t, S)$ is the default intensity, and $W_t^Q \in \mathbb{R}$ is a standard Wiener process generating the filtration $\mathbb{F} = \{\mathcal{F}_t, t \geq t_0\}$. Carr and Linetsky (2006) assume that the stock price $S$ can either diffuse or jump to default, i.e. the time of default is simply given by

$$
\zeta = \tau_0 \wedge \tilde{\zeta},
$$

where $\tilde{\zeta}$ is the first jump time of a doubly-stochastic Poisson process with intensity $\lambda(t, S)$, while

$$
\tau_0 := \inf \{t > t_0 : S_t = 0\}
$$

is the first passage time (through diffusion) of the stock price to the bankruptcy level.

Under the JDCEV framework, Nunes (2009) decomposes the American-style option price into a similar contract that assumes no default (until the earliest between the maturity date and the early exercise date) and a recovery component. However, for the American-style put the solution provided by Nunes (2009, Equation 56) for such recovery component is wrong. Section II corrects the pricing solution of Nunes (2009, Equation 56) while Section
III identifies the nonlinear integral equation satisfied by the first hitting time density through the early exercise boundary and under the JDCEV model. Finally, Section IV summarizes the changes made to Nunes (2009, Section VII) and recomputes Nunes (2009, Table 5).

II. Corrected Pricing Solutions

Under the JDCEV model, and assuming that $\zeta > t_0$, the time-$t_0$ value of an American put (if $\phi = 1$) or call (if $\phi = -1$) on the stock price $S$, with strike price $K$, and with maturity date $T \geq t_0$ is represented by Nunes (2009, Equation 53), i.e.:

$$V_{t_0}(S,K,T;\phi) = \sup_{\tau \in \mathcal{T}} \left\{ \mathbb{E}_{\mathbb{Q}} \left[ e^{-\int_{t_0}^{T \vee \tau} r(l) dl} (\phi K - \phi S_{T \wedge \tau})^+ \mathbb{1}_{\{\zeta > T \wedge \tau\}} \big| \mathcal{G}_{t_0} \right] ight\} + \mathbb{E}_{\mathbb{Q}} \left[ e^{-\int_{t_0}^{\zeta} r(l) dl} (\phi K)^+ \mathbb{1}_{\{\zeta \leq T \wedge \tau\}} \big| \mathcal{G}_{t_0} \right\},$$

where $\mathcal{T}$ is the set of all stopping times (taking values in $[t_0, \infty)$) for the enlarged filtration $\mathcal{G} = \{\mathcal{G}_t : t \geq t_0\}$, with $\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{D}_t$.\(^1\)

The second term on the right-hand side of equation (4) is zero for an American-style call option; for an American put, it corresponds to a recovery payment equal to the strike $K$ at the default time $\zeta$ (and as long as the default event precedes both the expiry and early exercise dates). The error in Nunes (2009) concerns the evaluation of this recovery value for the American-style put: Nunes (2009, Equation 56) wrongly assumes that $\{\zeta \leq T \wedge \tau\} = \{\zeta \leq T\}$, i.e. evaluates the recovery component associated not to an American-style but rather a European-style put option. However, the recovery component of an American-style put must be worth less than the one associated to the corresponding European-style contract because there can be no default after the (possible) early exercise event.\(^2\)

To correct Nunes (2009, Equation 56), and following, for instance, Carr and Linetsky (2006, Equations 3.2 and 3.4), equation (4) can be rewritten in terms of the restricted filtration $\mathcal{D} = \{\mathcal{D}_t : t \geq t_0\}$ is the filtration generated by the default indicator process $\mathcal{D}_t = \mathbb{1}_{(t > \zeta)}$.\(^1\)

\(^1\) Note that Nunes (2009) assumes that the recovery value is paid at the default time but Carr and Linetsky (2006, Equation 3.11) considers also the possibility of recovery only at the maturity date of the option (for European-style puts). For American-style options, this latter case would be even simpler to model.\(^2\)
filtration $\mathbb{F}$ as long as the short-term interest rate is replaced by an intensity-adjusted short-rate:

\[
V_t(0, S, K, T; \phi) = \sup_{\tau \in T} \left\{ \mathbb{E}_Q \left[ e^{-\int_0^{\tau \land T} (r(l) + \lambda(l,S))dl} (\phi K - \phi S_{\tau \land T})^+ \mathbb{1}_{\{\tau_0 > T \land \tau\}} \bigg| \mathcal{F}_t \right] \right. + (\phi K)^+ \mathbb{E}_Q \left[ \int_0^{\tau \land T} e^{-\int_0^v (r(l) + \lambda(l,S))dl} \lambda(v, S) \mathbb{1}_{\{\tau_0 > v\}} dv \bigg| \mathcal{F}_t \right] \right\}.
\]

Moreover, since $S$ behaves as a pure diffusion process with respect to the filtration $\mathbb{F}$, Detemple and Tian (2002, Propositions 1 and 2) show that there exists (at each time $t \in [t_0, T]$) a critical asset price $E_t$ below (above) which the American-style put (call) price equals its intrinsic value and, therefore, early exercise should occur. Consequently, and representing the first passage time of the underlying asset price $S$ to its early exercise boundary $\{E_t, t_0 \leq t \leq T\}$ by Nunes (2009, Equation 5), i.e.

\[
\tau_e := \inf \{ t \geq t_0 : S_t = E_t \},
\]
equation (5) can be restated as:

\[
V_t(0, S, K, T; \phi) = V_{t_0}^0 (S, K, T; \phi) + V_{t_0}^D (S, K, T; \phi),
\]

where

\[
V_{t_0}^0 (S, K, T; \phi) = \mathbb{E}_Q \left[ e^{-\int_0^{\tau_e \land T} (r(l) + \lambda(l,S))dl} (\phi K - \phi S_{\tau_e \land T})^+ \mathbb{1}_{\{\tau_0 > T \land \tau_e\}} \bigg| \mathcal{F}_{t_0} \right]
\]
corresponds to Nunes (2009, Equation 55) i.e. to the American option price conditional on no default (before the expiry and early exercise dates), and

\[
V_{t_0}^D (S, K, T; \phi) = (\phi K)^+ \mathbb{E}_Q \left[ \int_0^{\tau_e \land T} e^{-\int_0^v (r(l) + \lambda(l,S))dl} \lambda(v, S) \mathbb{1}_{\{\tau_0 > v\}} dv \bigg| \mathcal{F}_{t_0} \right]
\]
represents the present value of the recovery payment at the default time associated to the American-style put.

Nunes (2009, Equation 57 and Proposition 7) correctly decomposes the American option value conditional on no default (8) into the corresponding European option value (conditional on no default until the maturity date) and an early exercise premium, i.e.:

\[
V_{t_0}^0 (S, K, T; \phi) = v_{t_0}^0 (S, K, T; \phi) + EEP_{t_0}^0 (S, K, T; \phi),
\]
where

\[ v^0_{t_0} (S, K, T; \phi) = e^{-\int_{t_0}^{T} r_t dt} \mathbb{E}_Q \left[ (\phi K - \phi S_T)^+ \mathbb{1}_{\{T > \tau\}} \right | \mathcal{F}_{t_0} \]  

corresponds to Carr and Linetsky (2006, Equations 3.8 and 3.10), and

\[ EEP^0_{t_0} (S, K, T; \phi) = \int_{t_0}^{T} e^{-\int_{t_0}^{u} r_t dt} \left[ (\phi K - \phi E_u)^+ - v^0_u (E, K, T; \phi) \right] S P (t_0, u) \mathbb{Q} (\tau_e \in du | \mathcal{F}_{t_0}) , \]  

with

\[ SP (t_0, u) := \mathbb{E}_Q \left[ e^{-\int_{t_0}^{u} \lambda(t,S) dt} \mathbb{1}_{\{\tau_0 > u\}} \right | \mathcal{F}_{t_0} \]  

representing the risk-neutral probability of surviving beyond time \( u > t_0 \). However, the recovery component (9) must not be given by Nunes (2009, Equation 56) but, instead, must be also decomposed into an European-style recovery value and an additional (negative) early exercise premium component. Next proposition provides such decomposition and corrects Nunes (2009, Equation 56).

**Proposition 1** Under the JDCEV model, and assuming that \( \zeta > t_0 \), then

\[ V^D_{t_0} (S, K, T; \phi) = v^D_{t_0} (S, K, T; \phi) - EEP^D_{t_0} (S, K, T; \phi) , \]  

where

\[ v^D_{t_0} (S, K, T; \phi) = (\phi K)^+ \mathbb{E}_Q \left[ \int_{t_0}^{T} e^{-\int_{t_0}^{u} (r_t + \lambda(t,S)) dt} \lambda(v, S) \mathbb{1}_{\{\tau_0 > u\}} dv \right | \mathcal{F}_{t_0} \]  

is the recovery (at the default time) component of an European-style option contract, as given in Carr and Linetsky (2006, Equation 3.4), and

\[ EEP^D_{t_0} (S, K, T; \phi) = \int_{t_0}^{T} e^{-\int_{t_0}^{u} r_t dt} v^D_u (E, K, T; \phi) S P (t_0, u) \mathbb{Q} (\tau_e \in du | \mathcal{F}_{t_0}) \]  

is the early exercise component of the recovery value.

**Proof.** Since \( \mathbb{1}_{\{\tau_e > T\}} = 1 - \mathbb{1}_{\{\tau_e < T\}} \), equation (9) can be rewritten as:

\[ V^D_{t_0} (S, K, T; \phi) = (\phi K)^+ \mathbb{E}_Q \left[ \int_{t_0}^{T} e^{-\int_{t_0}^{u} (r_t + \lambda(t,S)) dt} \lambda(v, S) \mathbb{1}_{\{\tau_0 > v, \tau_e > T\}} dv \right | \mathcal{F}_{t_0} \]  

\[ + (\phi K)^+ \mathbb{E}_Q \left[ \int_{t_0}^{\tau_e} e^{-\int_{t_0}^{u} (r_t + \lambda(t,S)) dt} \lambda(v, S) \mathbb{1}_{\{\tau_0 > v, \tau_e < T\}} dv \right | \mathcal{F}_{t_0} \]  

\[ = v^D_{t_0} (S, K, T; \phi) - EEP^D_{t_0} (S, K, T; \phi) , \]
i.e. as in equation (14), where

$$
EEP_{t_0}^D (S, K, T; \phi) = (\phi K) + \mathbb{E}_Q \left[ \int_{t_0}^{T} e^{-\int_{t_0}^{u} (r_1 + \lambda(l,S))dl} \lambda(v,S) \mathbb{I}_{\{\tau_0 > v, \tau_e < T\}} dv \bigg| \mathcal{F}_{t_0} \right] 
$$

$$
= (\phi K) + \mathbb{E}_Q \left[ \int_{t_0}^{T} e^{-\int_{t_0}^{u} (r_1 + \lambda(l,S))dl} \lambda(v,S) \mathbb{I}_{\{\tau_0 > v, \tau_e < T\}} dv \bigg| \mathcal{F}_{t_0} \right] 
$$

(17)

Taking advantage of the Markovian nature of the underlying price process \(S\), the expectation on the right-hand side of equation (17) can be written as a convolution against the density of the first passage time \(\tau_e\), yielding

$$
EEP_{t_0}^D (S, K, T; \phi) = \int_{t_0}^{T} e^{-\int_{t_0}^{u} (r_1 + \lambda(l,S))dl} \lambda(v,S) \mathbb{I}_{\{\tau_0 > v, \tau_e < T\}} dv \bigg| \mathcal{F}_{t_0} \right] 
$$

where the last line corresponds to equation (16), and follows from equations (13) and (15).

Since \(\lambda(t, S) \geq 0\), for \(t \in [t_0, T]\), Proposition 1 implies that

$$
V_{t_0}^D (S, K, T; \phi) \leq v_{t_0}^D (S, K, T; \phi).
$$

Hence, and as expected, Nunes (2009, Equation 56) overvalues the recovery component of the American-style put since it simply corresponds to equation (15).

### III. The First Passage Time Density

Under the geometric Brownian motion or the CEV process, the optimal stopping time \(\tau_e\) is recovered by solving the nonlinear integral equation of Nunes (2009, Equation 35) through the standard partition method proposed by Park and Schuurmann (1976). However, under
the JDCEV model, the early exercise event only occurs at the stopping time \( \tau_e \) if \( \tau_e < \zeta \). Therefore, under the JDCEV model, the density of the first passage time \( \tau_e \) solves not Nunes (2009, Equation 35) but rather the following nonlinear integral equation.

**Proposition 2** Assuming that the underlying asset price \( S_t \) follows a JDCEV process and considering that the optimal exercise boundary is a continuous function of time, the first passage time density of the underlying asset price to the moving exercise boundary is the implicit solution of the following nonlinear integral equation:

\[
G^{-\phi}(t_0, S_{t_0}; u, E_u) = \int_{t_0}^{u} G^{-\phi}(v, E_v; u, E_u) \ SP(t_0, v) \ Q(\tau_e \in dv | \mathcal{F}_{t_0}),
\]

for \( \phi S_{t_0} > \phi E_{t_0} \) and \( u \in [t_0, T] \), and where \( \phi = -1 \) for an American call or \( \phi = 1 \) for an American put, with

\[
G^{-\phi}(v, E_v; u, E_u) = e^{-\int_v^u \bar{b} dl} \left[ \frac{k^2(v, u; E_u)}{\tau(v, u)} \right]^{ \frac{1}{2|\beta|}} \Phi_{\phi} \left( -\frac{1}{2|\beta|}, \frac{k^2(v, u; E_u)}{\tau(v, u)} ; 2(1 + v_+), \frac{k^2(v, v; E_v)}{\tau(v, u)} \right),
\]

\[
v_+ := \frac{c + \frac{1}{2}}{|\beta|},
\]

\[
\tau(v, u) := \int_v^u \alpha_e^2 e^{-2|\beta| \int_v^u \alpha_v dl} ds,
\]

\[
k(v, u; S) := \frac{1}{|\beta|} \mathcal{E}_{|\beta|} e^{-|\beta| \int_v^u \alpha_v dl}
\]

and

\[
\alpha_t := r_t - q_t + b_t.
\]

**Proof.** By the law of total probability,

\[
Q(\phi S_u \leq \phi E_u, \zeta > u | \mathcal{G}_t) = Q(\phi S_u \leq \phi E_u, \zeta > u, \tau_e \leq u | \mathcal{G}_t),
\]

The function \( \Phi_{\phi}(p, y; v, \lambda) := \mathbb{E}^{X^2(v, \lambda)}(X^p \mathbb{1}_{\{\phi X > \phi y\}}) \) represents, for \( \phi \in \{-1, 1\} \), the truncated \( p \)-th moments of a noncentral chi-square random variable \( X \) with \( v \) degrees of freedom and noncentrality parameter \( \lambda \), as defined in Carr and Linetsky (2006, Equations 5.11 and 5.12).
since \( Q(\phi S_u \leq \phi E_u, \zeta > u, \tau_e > u | \mathcal{G}_{t_0}) = 0 \).

Starting with the left-hand side of equation (24) and using Carr and Linetsky (2006, Equation 3.2), then

\[
Q(\phi S_u \leq \phi E_u, \zeta > u | \mathcal{G}_{t_0}) = \mathbb{E}_Q \left( \mathbb{I}_{\{\phi S_u \leq \phi E_u, \zeta > u\}} | \mathcal{G}_{t_0} \right)
\]

\[
= \mathbb{E}_Q \left( e^{-\int_{t_0}^{u} \lambda(S,t)dt} \mathbb{I}_{\{\phi S_u \leq \phi E_u, \tau_0 > u\}} | \mathcal{F}_{t_0} \right).
\]

(25)

Moreover, Carr and Linetsky (2006, Proposition 5.4) allows equation (25) to be rewritten as

\[
Q(\phi S_u \leq \phi E_u, \zeta > u | \mathcal{G}_{t_0})
\]

\[
= e^{-\int_{t_0}^{u} b(t)dt} \mathbb{E}_{k(t_0,t_0;S_0)}^{(v_+)} \left[ \left( \frac{R_{\tau(t_0,u)}}{k(t_0,t_0;S_0)} \right)^{-\frac{1}{|\beta|}} \mathbb{I}_{\{\phi \int_{t_0}^{u} \alpha ds | R_{\tau(t_0,u)} | |\beta| \leq \phi E_u \}} \right]
\]

\[
\mathbb{E}_{k(t_0,t_0;S_0)}^{(v_+)} \left[ \left( \frac{R_{\tau(t_0,u)}}{\tau(t_0,u)} \right)^{-\frac{1}{2|\beta|}} \mathbb{I}_{\{\phi \frac{R_{\tau(t_0,u)}}{\tau(t_0,u)} | \beta | \leq \phi E_u \}} \right] \]

\[
= e^{-\int_{t_0}^{u} b(t)dt} \mathbb{E}_{k(t_0,t_0;S_0)}^{(v_+)} \left[ \frac{k^2(t_0,t_0;S_0)}{\tau(t_0,u)} \right]^{-\frac{1}{2|\beta|}}
\]

(26)

where the expectation is taken with respect to the law of a Bessel process \( \{R_{\tau(t_0,u)}; u \geq t_0\} \) of index \( v_+ \) and started at \( R_{\tau(t_0,t_0)} = k(t_0,t_0;S_0) \). Finally, using Carr and Linetsky (2006, Equations 5.11 and 5.12), and since the process \( \frac{R_{\tau(t_0,u)}}{\tau(t_0,u)} \) follows a noncentral chi-square law with \( 2(1 + v_+) \) degrees of freedom and non-centrality parameter \( \frac{k^2(t_0,t_0;S_0)}{\tau(t_0,u)} \), equation (26) yields

\[
Q(\phi S_u \leq \phi E_u, \zeta > u | \mathcal{G}_{t_0}) = e^{-\int_{t_0}^{u} b(t)dt} \Phi_{-\phi} \left( -\frac{1}{2|\beta|}; \frac{k^2(t_0,t_0;S_0)}{\tau(t_0,u)} \right)
\]

\[
\left[ \frac{k^2(t_0,t_0;S_0)}{\tau(t_0,u)} \right]^{-\frac{1}{2|\beta|}}
\]

(27)

Concerning the right-hand side of equation (24),

\[
Q(\phi S_u \leq \phi E_u, \zeta > u, \tau_e \leq u | \mathcal{G}_{t_0}) = \mathbb{E}_Q \left( \mathbb{I}_{\{\phi S_u \leq \phi E_u, \zeta > u, \tau_e \leq u\}} | \mathcal{G}_{t_0} \right)
\]

\[
= \mathbb{E}_Q \left( e^{-\int_{t_0}^{u} \lambda(S,t)dt} \mathbb{I}_{\{\phi S_u \leq \phi E_u, \tau_0 > u, \tau_e \leq u\}} | \mathcal{F}_{t_0} \right),
\]

(28)

where the last line follows again from Carr and Linetsky (2006, Equation 3.2). Since \( S_t \) behaves as a pure and Markovian diffusion process with respect to the filtration \( \mathbb{F} \), equation
(28) can be rewritten in terms of the convolution between the densities of the first passage time $\tau_e$ and of the random vector $(S_u, \tau_0)$:

$$
\mathbb{Q}\left( \phi S_u \leq \phi E_u, \zeta > u, \tau_e \leq u \mid \mathcal{G}_0 \right) = \int_0^u \mathbb{E}\mathbb{Q}\left( e^{-\int_0^u \lambda(S,l)dt} \mathbb{1}_{\{\phi S_u \leq \phi E_u, \tau_e > u, S_v = E_v \}} \mid \mathcal{F}_0 \right) \mathbb{Q}(\tau_e \in dv \mid \mathcal{F}_0) = \int_0^u \mathbb{E}\mathbb{Q}\left( e^{-\int_0^u \lambda(S,l)dt} \mathbb{1}_{\{\phi S_u \leq \phi E_u, \inf_{t_0 \leq t \leq u} (S_l) > 0 \}} S_v = E_v \mid \mathcal{F}_0 \right) \mathbb{Q}(\tau_e \in dv \mid \mathcal{F}_0),
$$

(29)

where the last equality follows from the tower law for conditional expectations. Comparing the inner expectation on the right-hand side of equation (29) with equation (25), and using equation (27), then

$$
\mathbb{E}\mathbb{Q}\left( e^{-\int_v^u \lambda(S,l)dt} \mathbb{1}_{\{\phi S_u \leq \phi E_u, \inf_{t_0 \leq t \leq u} (S_l) > 0 \}} S_v = E_v \right) = G_{-\phi}(v, E_v; u, E_u),
$$

and, therefore, equation (29) becomes

$$
\mathbb{Q}\left( \phi S_u \leq \phi E_u, \zeta > u, \tau_e \leq u \mid \mathcal{G}_0 \right) = \int_0^u G_{-\phi}(v, E_v; u, E_u) \mathbb{E}\mathbb{Q}\left( e^{-\int_0^u \lambda(S,l)dt} \mathbb{1}_{\{\inf_{t_0 \leq t \leq v} (S_l) > 0 \}} \mid \mathcal{F}_0 \right) \mathbb{Q}(\tau_e \in dv \mid \mathcal{F}_0) = \int_0^u G_{-\phi}(v, E_v; u, E_u) SP(t_0, v) \mathbb{Q}(\tau_e \in dv \mid \mathcal{F}_0),
$$

(30)

where the last line follows from definition (13).

Combining equations (24), (27), and (30), equation (18) arises immediately.

**IV. Changes to the Original Paper**

In summary, the first correction that shall be done to the pricing solutions proposed in Nunes (2009, Section VII) is to replace Nunes (2009, Equation 56) by equation (14). Combining equations (7), (10), and (14), this is equivalent to rewrite the American-style option price under the JDCEV model as

$$
V_{t_0}(S, K, T; \phi) = \left[ v_{t_0}^0 (S, K, T; \phi) + v_{t_0}^D (S, K, T; \phi) \right] + EEP_{t_0} (S, K, T; \phi),
$$

(31)
where

\[ EEP_{t_0} (S, K, T; \phi) = EEP^0_{t_0} (S, K, T; \phi) - EEP^D_{t_0} (S, K, T; \phi) \]

represents the overall early exercise premium while the first two terms on the right-hand side of equation (31) are simply the price of an European-style option whose recovery component assumes payment at the default time.

Since the time path \( \{E_t, t_0 \leq t \leq T\} \) of critical asset prices is not known ex ante, to use equation (31), we must first parameterize such early exercise boundary, and maximize (with respect to those parameters) not Nunes (2009, Equation 60) but the overall early exercise premium (32). For this purpose, and as a second correction, the density of the optimal passage time \( \tau_e \) shall be recovered by solving—through the standard partition method proposed by Park and Schuurmann (1976)—the nonlinear integral equation (18) instead of Nunes (2009, Equation 35).

Using the above two mentioned corrections, Table 1 recomputes Nunes (2009, Table 5).
References


Table 1: American Put Options under the JDCEV Model, with $\bar{\beta} = -0.5$, $c = 0.5$, $b = 0.02$, $S_{t_0} = $100 and $T-t_0 = 5$ years

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<td>40.227</td>
<td>45.753</td>
<td>10.796</td>
<td>16.322</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>5.134</td>
<td>26.532</td>
<td>31.666</td>
<td>9.991</td>
<td>15.124</td>
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<tr>
<td></td>
<td>90</td>
<td>7.496</td>
<td>25.633</td>
<td>33.129</td>
<td>1.849</td>
<td>9.345</td>
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<tr>
<td>$q = 7%$</td>
<td>100</td>
<td>10.001</td>
<td>28.481</td>
<td>38.482</td>
<td>2.516</td>
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</tr>
<tr>
<td></td>
<td>110</td>
<td>12.891</td>
<td>31.329</td>
<td>44.220</td>
<td>3.332</td>
<td>16.223</td>
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<tr>
<td>$a = 3$</td>
<td>120</td>
<td>16.148</td>
<td>34.177</td>
<td>50.325</td>
<td>4.314</td>
<td>20.462</td>
</tr>
</tbody>
</table>

Table 1 values American-style put options under a time-homogeneous jump to default extended CEV (JDCEV) model. The third column contains European-style put prices, conditional on no default, as given by equation (11). The fourth column reports the recovery component of the European put contract, which is computed from Carr and Linetsky (2006, Equation 5.15), and the fifth column yields the sum of the previous two components. The early exercise premium components (sixth and eighth columns) are computed through equations (12) and (16) and via Proposition 2, using a polynomial specification for the exercise boundary with four degrees of freedom. The seventh, ninth and tenth columns are given by equations (10), (14) and (7), respectively. For comparison purposes, the last column presents the prices of the American-style put options computed through the static hedge approach proposed by Ruas et al. (2013).