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Appendix to "The impact of regulation fair disclosure: trading costs and information asymmetry," Eleswarapu, Thompson, and Venkataraman.

Testing Joint Inequality Restrictions

This appendix contrasts the testing of joint inequality restrictions with the more traditional test of a null hypothesis that there is no effect. Chart A shows a two dimensional parameter space in which the hypothesis of cost decrease requires the parameters to be in the negative quadrant, while the hypothesis of cost increase restricts the parameters to the positive quadrant. Testing a joint inequality restriction involves assessing the probability that the two parameter estimates actually observed could have occurred when the two true parameters are within the region allowed by each of the hypotheses. Clearly, if both estimates are within the restricted region of one of the hypotheses, a conservative test would not indicate rejection of that hypothesis. However, if one or both of the parameter estimates lie outside the restricted region, then the distance to the restricted region can be used to infer the probability that the estimates could have occurred by chance when the hypothesis is true.

A conservative test would measure the distance to the closest point in the restricted region, assessing the probability that the estimates could have occurred by chance when the true parameters are at the closest point. This is a conservative test because the probability of observing the estimates would be lower if the true parameters were at any further point in the restricted region.

The traditional quadratic form used to construct a Chi-squared test of joint restrictions (often called the Wald statistic) can be used but inferences must account for the probability that the estimates are on the wrong side of the inequality. This accounting is achieved by comparing the statistic to a weighted average of Chi-squared variates, some having lower degrees of freedom than the number of inequalities as described in Wolak (1989).

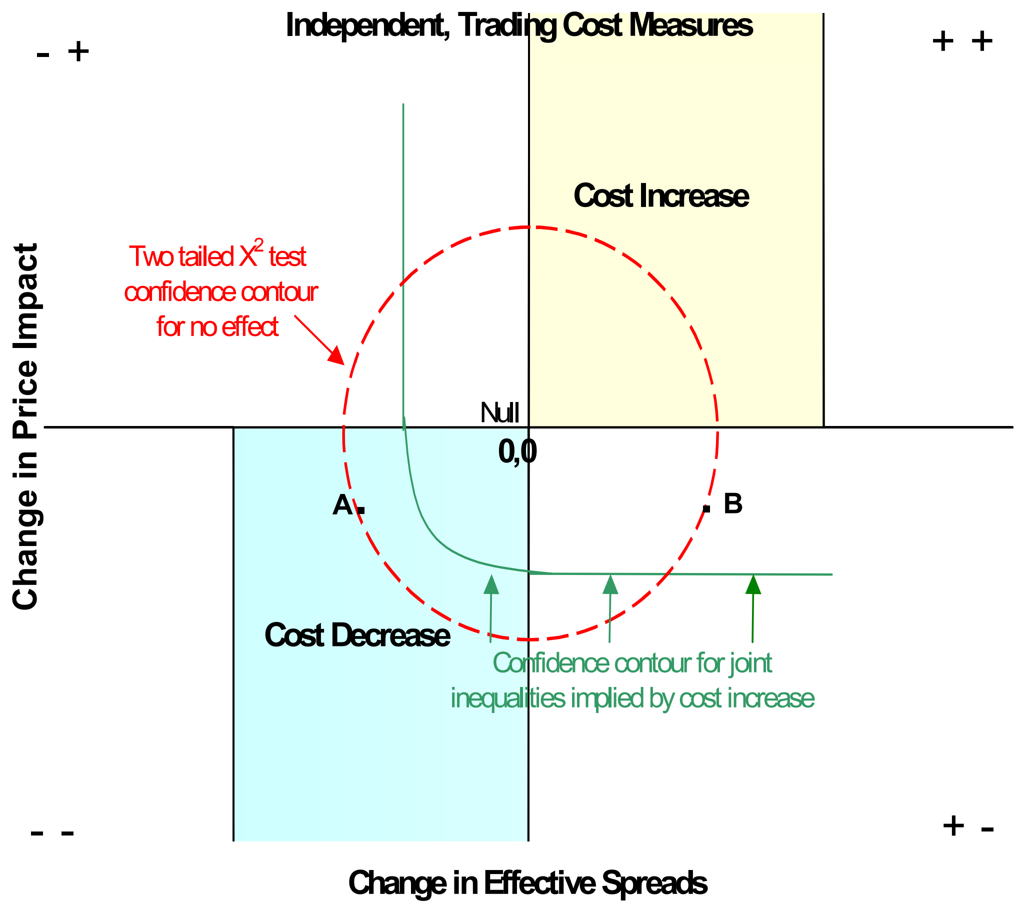
To clarify the intuition, consider the two points A and B in Chart A. They are both the same distance from the 0,0 point. The traditional Chi-squared test with two degrees of freedom would indicate a particular confidence for either A or B. But clearly, under the hypothesis of a cost increase, A is less probable than B because B consists of one estimate that is the same as A and one estimate within the restricted region consistent with a cost increase. Since the probability of B is greater under the hypothesis of a cost increase, the probability of A must be less than indicated by the traditional Ch-square with two degrees of freedom. The reason that the Chi-square with two degrees of freedom overstates the confidence is because it fails to account for the lower probability that estimates will be in the direction of A than in the direction of B when the hypothesis of a cost increase is true.

If we draw a confidence contour for a cost increase with the same probability as the confidence contour for the Chi-square with two degrees of freedom it would look as indicated in the Chart, lying inside the contour for the region in which the probability is lowest of observing parameter estimates. By controlling for the lower probability, the joint test takes the intuition of a one-tailed test for univariate inequality restrictions. When constructing a one-tailed test, the same z-statistic is used that would be used for a two tailed test but accounting for the one tailed direction of violations of the inequality allows an inference of a lower probability when the estimates are in the wrong direction.

As shown in Wolak (1989), for two inequalities, an exact (asymptotic) test is achieved by comparing the Wald statistic to a weighted average of Chi-squared variates with weights of 0.5 for $X^2(1)$ and $0.5\cdot(1\text{-ACOS}(\rho/\pi))$ for $X^2(2)$, where ρ is the correlation between the estimates. Referring back to the Chart, the significance of A would be found by finding the probability of a $X^2(1)$ with magnitude A, assigning it the weight of .5, then finding the probability of a $X^2(2)$ with magnitude A, assigning it the weight indicated by the formula and adding these together. For cases where only one inequality is violated, as at point B, only the distance of the violation is measured and compared against the weighted average of Chi-squared variates. This is captured by the confidence contours for joint inequalities as shown.

Chart A. Impact of Regulation FD on Trading Costs:

Parameter Space for the Change in Two, Independent, Trading Cost Measures



A and B represent two sets of parameter estimates, which have the same probability under the null hypothesis of no effect. A is less probable than B under the inequality restriction that the changes in both trading cost measures are non-negative.