Horses and Rabbits? Trade-Off Theory and Optimal Capital Structure

Appendix A

In this appendix we develop the dynamic model in more detail. To obtain $G(T), H(T)$, and $I(T)$[1] in equations (4), (5), and (6) of the main text, we require the first passage time density function. To compute it, we first define

$$x(t) \equiv \log\left(\frac{V(t)}{F_L e^{-g(T-t)}}\right). \tag{A.1}$$

One application of Ito's lemma under the risk-neutral measure yields

$$dx = \left(r - \delta - g - \sigma^2/2\right)dt + \sigma\, dZ^Q(t). \tag{A.2}$$

Consequently, $x(t)$ is a Brownian motion with drift $m \equiv r - \delta - g - \sigma^2/2$ and diffusion σ, starting at $x_0 = \log\left(\frac{V(0)}{F_L e^{-gT}}\right)$. From Ingersoll (1987), the first-passage time density function $f(t)$ for crossing the origin is given by

$$f(t) = \frac{x_0}{\sigma\, t^{3/2}}\, n\left(\frac{x_0 + mt}{\sigma\, t^{1/2}}\right), \tag{A.3}$$

where $n(\bullet)$ is the standard normal density function. Now, lengthy, but straightforward calculations yield[2]

$$G(T) = N\left[h_1(T)\right] + \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-2a} N\left[h_2(T)\right], \tag{A.4}$$

$$H(T) = \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a+z} N\left[q_1(T)\right] + \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a-z} N\left[q_2(T)\right], \tag{A.5}$$

$$I(T) = \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a+\bar{z}} N\left[\bar{q}_1(T)\right] + \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a-\bar{z}} N\left[\bar{q}_2(T)\right], \tag{A.6}$$

where

[1] For simplicity, we omit the other arguments of these functions.
[2] Explicit derivation is available upon request.

$$h_1(T) \equiv \left(\frac{-x_0 - a\sigma^2 T}{\sigma\sqrt{T}} \right), \qquad h_2(T) \equiv \left(\frac{-x_0 + a\sigma^2 T}{\sigma\sqrt{T}} \right),$$

$$q_1(T) \equiv \left(\frac{-x_0 - z\sigma^2 T}{\sigma\sqrt{T}} \right), \qquad q_2(T) \equiv \left(\frac{-x_0 + z\sigma^2 T}{\sigma\sqrt{T}} \right), \tag{A.7}$$

$$\bar{q}_1(T) \equiv \left(\frac{-x_0 - \bar{z}\sigma^2 T}{\sigma\sqrt{T}} \right), \qquad \bar{q}_2(T) \equiv \left(\frac{-x_0 + \bar{z}\sigma^2 T}{\sigma\sqrt{T}} \right),$$

$$a \equiv \frac{\left(r - \delta - g - \sigma^2/2\right)}{\sigma^2}, \qquad z \equiv \frac{\left[\left(a\sigma^2\right)^2 + 2r\sigma^2\right]^{1/2}}{\sigma^2}, \qquad \bar{z} \equiv \frac{\left[\left(a\sigma^2\right)^2 + 2\left(r - g\right)\sigma^2\right]^{1/2}}{\sigma^2}.$$

In these expressions, $N(\bullet)$ is the cumulative standard normal distribution function.

Given $G(T), H(T),$ and $I(T),$ the values of the debt, bankruptcy costs, and tax shields of the current debt are given by equations (8), (11), and (13) respectively. The total firm value in the static model, when debt is issued only once by the firm, is given by the value of the firm's unlevered assets plus the tax shields of debt (equation (13)) minus the bankruptcy costs (equation (11))

$$TV_L(0) = V(0) + TB_L(0) - BC_L(0) \; . \tag{A.8}$$

We now turn our attention to the dynamic model. In this model the firm repeatedly and optimally issues *T*-year maturity debt until it goes bankrupt. Obviously, the optimal coupon for the new issues will depend on the firm value when the future debt is issued. We note, however, the following scaling property: If the optimal coupon of the first (initial) debt issue is C_L, then the optimal coupon in future issues will be scaled by the ratio of the asset value $V(t)$ when the new debt is issued to the initial asset value $V(0)$. The reason for this is that at time t the firm is identical to itself at time zero, except that it is $V(t)/V(0)$ as large because the asset value follows a proportional process (geometric Brownian motion). Therefore, if no bankruptcy has occurred by the time that the initial debt matures at T, the optimal coupon of the new debt will be $C_L\left(V(T)/V(0)\right)$. Now, if bankruptcy occurs at $t^* < T$, the asset value will be $F_L e^{-g\left(T-t^*\right)}$. We allow the debtholders to become the new shareholders, and they optimally lever the remaining asset value (1-α_{BC}) $F_L e^{-g\left(T-t^*\right)}$ after the bankruptcy process consumes α_{BC} $F_L e^{-g\left(T-t^*\right)}$. Thus, the optimal coupon after bankruptcy reorganization is $C_L \dfrac{(1-\alpha_{BC})F_L e^{-g(T-t^*)}}{V(0)}$. In fact, all future issues of debt will be scaled by the ratio of the firm's asset value when the new debt is issued to its asset value when the old debt is issued.

Even though only the current issue of debt is outstanding at time zero, the tax shields and bankruptcy costs reflect all expected future debt issues. Let $TB_L^{Dynamic}(0)$ denote the total tax shields in the dynamic model. The scaling property discussed in the previous paragraph implies that the total tax shields at time T will be $TB_L^{Dynamic}(0)\left(V(T)/V(0)\right)$ if no bankruptcy has yet occurred and it will be $TB_L^{Dynamics}(0)\dfrac{(1-\alpha_{BC})F_L e^{-g(T-t^*)}}{V(0)}$ if bankruptcy occurs at t^*. Risk-neutral valuation implies that the initial *total* tax shield, $TB_L^{Dynamic}(0)$, is equal to the tax shield from the initial debt plus the conditional discounted risk-neutral expected total tax shield at time T plus the conditional discounted risk-neutral expected total tax shield if bankruptcy occurs at $t^* < T$. If we now let $TB_L(0)$ denote the tax shield from the initial issue of debt, we have[3]

$$
\begin{aligned}
TB_L^{Dynamic}(0) &= TB_L(0) + \int_{F_L}^{\infty} e^{-rT}\,\frac{V(T)}{V(0)}TB_L^{Dynamic}(0)\,\rho\left(V(T)\right)dV(T) \\
&\quad + \int_0^T e^{-rt^*}\,\frac{(1-\alpha)F_L e^{-g(T-t^*)}}{V(0)}TB_L^{Dynamic}(0)\,f(t^*)\,dt^* \qquad \text{(A.9)} \\
&= TB_L(0) + \phi TB_L^{Dynamic}(0),
\end{aligned}
$$

[3] For simplicity, we omit the other arguments of $\rho\left(V(T)\right)$ and $f(t^*)$.

where ϕ is defined in equation (15). Solving for $TB_L^{Dynamic}(0)$, we obtain

$$TB_L^{Dynamic}(0) = \frac{TB_L(0)}{1-\phi}. \tag{A.10}$$

The total tax shields have an intuitive series expansion. Each term in the expansion

$$TB_L^{Dynamic}(0) = TB_L(0)\left(1+\phi+\phi^2+\phi^3+\cdots\right) \tag{A.11}$$

represents the present value of the tax shields from the debt issue in each succeeding period.

To find ϕ, we require the conditional distribution of $V(T)$ such that the firm has not gone bankrupt at time T. Again, from Ingersoll (1987) we have the following conditional density function for $V(T)$:

$$\rho(V(T)) = \frac{1}{V(T)\sigma\sqrt{T}} n\left(\frac{x(T)-x_0-mT}{\sigma\sqrt{T}}\right) - \frac{e^{\frac{2mx_0}{\sigma^2}}}{V(T)\sigma\sqrt{T}} n\left(\frac{x(T)+x_0-mT}{\sigma\sqrt{T}}\right). \tag{A.12}$$

Using the above density function and the first passage time density $f(t^*)$ given in equation (A.3), tedious but straightforward derivations yield the following closed form solution for ϕ,

$$\phi = e^{-\delta T}\left(N(d_1) - \left(\frac{F_L e^{-gT}}{V(0)}\right)^{2\lambda} N(d_2)\right) + \frac{(1-\alpha_{BC})F_L e^{-gT}}{V(0)} I(T), \tag{A.13}$$

where $I(T)$ is given in equation (A.6), $\lambda = 1 + m/\sigma^2$ and

$$d_1 = \frac{-\log\left(F_L e^{-gT}/V(0)\right) + \left(r-\delta-g+\sigma^2/2\right)T}{\sigma\sqrt{T}},$$
$$d_2 = \frac{\log\left(F_L e^{-gT}/V(0)\right) + \left(r-\delta-g+\sigma^2/2\right)T}{\sigma\sqrt{T}}. \tag{A.14}$$

Similarly, the total bankruptcy costs in the dynamic model, $BC_L^{Dynamic}(0)$, are given by

$$BC_L^{Dynamic}(0) = \frac{BC_L(0)}{1-\phi}. \tag{A.15}$$

The total levered firm value, $TV_L^{Dynamic}(0)$, in the dynamic model equals the unlevered firm value $V(0)$, plus the total tax shields $TB_L^{Dynamic}$, less the total bankruptcy costs $BC_L^{Dynamic}(0)$,

$$TV_L^{Dynamic}(0) = V(0) + TB_L^{Dynamic}(0) - BC_L^{Dynamic}(0) = V(0) + \frac{TB_L(0) - BC_L(0)}{1-\phi}. \qquad (A.16)$$

The optimal capital structure is obtained by maximizing either the total firm value or the manager's utility.

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Appendix B

In this appendix we develop a dynamic counterpart to the static Leland (1994) model. In this model the firm optimally issues a perpetual callable bond with coupon rate C and principal P at time zero. This debt issue ceases to exist under either of two conditions. First, if the firm value $V(t)$ reaches a lower boundary V_B (which is an endogenously determined bankruptcy level) before an upper level V_U (which is a restructuring level chosen to maximize the initial total levered firm value), the firm goes bankrupt and the debt holders receive $(1-\alpha_{BC})V_B$ (where α_{BC} is the bankruptcy cost parameter). Second, if $V(t)$ reaches the upper boundary V_U before the lower level V_B, the debt is called at face value. At the same time, a new issue of another perpetual callable bond is optimally issued. Since the time when either boundary is first hit is random, the price of the debt at any point before either boundary is hit does not depend explicitly on time. It satisfies the following ordinary differential equation (ODE),

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r-\delta)VD_V - rD + C = 0. \tag{B.1}$$

The solution to this ODE is

$$D(V) = \frac{C}{r} + aV^{x_1} + bV^{x_2}, \tag{B.2}$$

where

$$\begin{aligned} x_1 &= \frac{-(r-\delta-\sigma^2/2) + \sqrt{(r-\delta-\sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2}, \\ x_2 &= \frac{-(r-\delta-\sigma^2/2) + \sqrt{(r-\delta-\sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2}, \end{aligned} \tag{B.3}$$

and a and b are determined by boundary conditions. Applying the two boundary conditions,

$$D(V_B) = (1-\alpha)V_B, \qquad D(V_U) = P, \tag{B.4}$$

we have

$$a = \left(V_U^{x_2}\left((1-\alpha)V_B - C/r\right) - V_B^{x_2}\left(P - C/r\right)\right)\big/\Sigma,$$
$$b = \left(V_B^{x_2}\left(P - C/r\right) - V_U^{x_2}\left((1-\alpha)V_B - C/r\right)\right)\big/\Sigma, \tag{B.5}$$

where $\Sigma = V_B^{x_1}V_U^{x_2} - V_U^{x_1}V_B^{x_2}$. As is typical in practice, we require the debt to be issued at par. Setting $D(V(0)) = P$, and solving for P, we have

$$D(V) = P = \frac{C}{r} + \left((1-\alpha_{BC})V_B - \frac{C}{r}\right)\frac{V_U^{x_2}V(0)^{x_1} - V_U^{x_1}V(0)^{x_2}}{V_U^{x_2}V_B^{x_1} - V_U^{x_1}V_B^{x_2} + V_B^{x_2}V(0)^{x_1} - V_B^{x_1}V(0)^{x_2}}. \tag{B.6}$$

Let $F(V)$ be the price of a contingent claim which in addition to a payoff at the boundaries, has claim to a cashflow E. Then $F(V)$ has the following solution,

$$F(V) = E/r + c_1V^{x_1} + c_2V^{x_2}, \tag{B.7}$$

where c_1 and c_2 are determined by the boundary conditions.

If we let $TB(V)$ represent the total tax shields (from current outstanding debt and all future debt issues), then the cashflow is $E = \tau\ C$, $TB(V_B) = 0$, and $TB(V_U) = TB(V)\left(V_U/V(0)\right)$. The last condition results from the scaling property discussed in Appendix A because the total tax shields at the restructuring point V_U should be $\left(V_U/V(0)\right)$ as large as it is at $V(0)$. Now, a straightforward derivation yields the following solution for $TB(V)$,

$$TB(V) = \tau C/r + hV^{x_1} + kV^{x_2}, \tag{B.8}$$

where

$$h = \left(-V_U^{x_2}\tau C/r - V_B^{x_2}\left(TB(V(0))V_U/V(0) - \tau C/r\right)\right)\big/\Sigma,$$
$$k = \left(-V_U^{x_1}\tau C/r - V_B^{x_1}\left(TB(V(0))V_U/V(0) - \tau C/r\right)\right)\big/\Sigma, \tag{B.9}$$

and $TB(V(0)) = \dfrac{tb(V(0))}{1-\phi}$, where

$$tb(V(0)) = \tau C/r\left(1 + \frac{\left(V(0)^{x_1}V_B^{x_2} - V_B^{x_1}V(0)^{x_2}\right) + \left(V_U^{x_1}V(0)^{x_2} - V(0)^{x_1}V_U^{x_2}\right)}{V_B^{x_1}V_U^{x_2} - V_U^{x_1}V_B^{x_2}}\right), \tag{B.10}$$

$$\phi = \frac{\left(V_B^{x_2}V(0)^{x_1} - V_B^{x_1}V(0)^{x_2}\right)V_U}{\left(V_B^{x_2}V_U^{x_1} - V_B^{x_1}V_U^{x_2}\right)V(0)}. \tag{B.11}$$

Similarly, the total bankruptcy costs are given by

$$BC(V) = fV^{x_1} + wV^{x_2}, \tag{B.12}$$

where

$$\begin{aligned} f &= \left(V_U^{x_2}\alpha_{BC}V_B - V_B^{x_2}BC(V(0))V_U/V(0)\right)/\Sigma, \\ w &= \left(V_B^{x_1}BC(V(0))V_U/V(0) - V_U^{x_1}\alpha_{BC}V_B\right)/\Sigma, \end{aligned} \tag{B.13}$$

and $BC(V(0)) = \dfrac{bc(V(0))}{1-\phi}$, where

$$bc(V(0)) = \alpha_{BC}V_B\left(\frac{V_U^{x_1}V(0)^{x_2} - V(0)^{x_1}V_U^{x_2}}{V_U^{x_1}V_B^{x_2} - V_B^{x_1}V_U^{x_2}}\right). \tag{B.14}$$

To prevent the firm from restructuring continuously, we introduce a proportional transaction cost for issuing new debt. The total transaction cost is

$$TC(V) = qV^{x_1} + sV^{x_2}, \tag{B.15}$$

where

$$q = -\frac{TC(V(0))V_UV_B^{x_2}}{V(0)\Sigma}, \qquad s = \frac{TC(V(0))V_UV_B^{x_1}}{V(0)\Sigma}, \tag{B.16}$$

and $TC(V(0)) = \dfrac{tc(V(0))}{1-\phi}$, $tc(V(0)) = \beta P$, where β is the transaction cost on each dollar issued and P is the principal of the initial debt.

The total value of the levered firm is the value of the unlevered firm V, plus the total tax shields $TB(V)$, less the total bankruptcy costs $BC(V)$, less the total transaction cost $TC(V)$,

$$TLV(V) = V + TB(V) - BC(V) - TC(V)\,. \tag{B.17}$$

The equity value after the debt is issued is

$$E(V) = TLV(V) - D(V) = V + (\tau - 1)C/r + (h - f - q - a)V^{x_1} + (k - w - s - b)V^{x_2}\,. \tag{B.18}$$

The limited liability of the equity is satisfied if $\partial E(V)/\partial V\,|_{V=V_B} = 0$ (which is a smooth-pasting condition). That is,

$$1 + x_1(h - f - q - a)V_B^{x_1 - 1} + x_2(k - w - s - b)V_B^{x_2 - 1} = 0\,. \tag{B.19}$$

Therefore, the firm maximizes $TLV(V(0))$, subject to the condition in equation (B.19).

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Appendix C

Model and Estimates for Utility Maximizing Risk-Averse Manager

Because agency considerations have been such a prominent feature of the capital structure literature, we also introduce agency conflicts into our analysis by calculating the value of the debt that, instead of maximizing the total value of the levered firm, maximizes the utility of a risk-averse manager. This appendix describes an expanded version of the model in the main text that incorporates decision-making by a risk-averse manager and reports implied optimal capital structures, from the manager's perspective, for the typical firm and for the individual firms examined in the main text.

C.I. A dynamic model of capital structure choice with a risk-averse manager

The model we use is based on Ju (1998, 2001). In this model, the firm issues debt with a maturity of T, which pays a continuous, constant (tax-deductible) coupon. The manager's wealth at time zero is divided between non-firm wealth and his stake in the firm, which consists of equity shares and standard European call options on the firm's shares, which expire at time T_u. The manager cannot sell or hedge his shares or options. For simplicity, it is assumed that the manager's non-firm wealth grows at the risk-free rate, r, and is therefore uncorrelated with the value of the manager's stake in the firm. The manager's utility is given by a CRRA utility function that is defined over his entire wealth. The value process of the firm's assets (that is, the value of the cash flows from operations) follows geometric Brownian motion.

The model is in continuous time with $0 < T_u < T$. At time zero the value of the firm's assets is $V(0)$. Without debt in its capital structure, the firm's capital consists of N_{NL} shares of stock with a total market value of $E_{NL}(0) = V(0)$.[1] The value of the firm's assets, $V(t)$, follows geometric Brownian

[1] The subscript NL refers to quantities when the firm is not leveraged (that is, when it does not have debt in its capital structure). The subscript L refers to quantities when the firm is leveraged.

motion described by:

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma dZ(t), \tag{C.1}$$

where μ and $\sigma > 0$ are constants and $Z(t)$ is a standard Wiener process. The firm liquidates assets at a rate of δ of the total value of the firm's assets, so that $\delta V(t)dt$ is equal to a time varying dividend $div(t)dt$ paid to equity holders over the time interval dt:

$$\delta V(t)dt = div(t)dt. \tag{C.2}$$

The value of δ is specified exogenously as a model parameter.

The manager makes a capital structure decision at time zero, which consists of choosing a level of debt that maximizes his expected utility at time T_u. The debt has a face value of F_L and has a market value when it is issued at time zero of $D_L(0)$. The debt pays a coupon at a constant annualized rate C_L which is set so that the debt is priced at par, that is, $F_L = D_L(0)$. The firm deducts its coupon payments from its taxes at an effective rate τ, and the tax shields of the debt at time zero have a value of $TB_L(0)$. The debt has a protective covenant which specifies that if the asset value, at any time during the life of the debt $[0,T]$, decreases to an exponential boundary, the firm is forced into bankruptcy.[2] Besides offering tractability, this default boundary form contains several default triggering mechanisms as special cases, e.g., the positive net-worth protected debt case in Leland (1994) and the constant default boundary case of Longstaff and Swartz (1995). When default occurs, the stock becomes worthless and the debtholders

[2] As in Black and Cox (1976), our bankruptcy boundary increases through time at an exponential rate until it reaches the face value of the debt at the time that the debt matures. The boundary is intended to act somewhat like covenants in bond indenture agreements that give bondholders the right to seize assets when they are in danger of being lost. Huang and Huang (2003) discuss that firms often continue to operate even when their asset values fall below the face value of outstanding debt. On the other hand, at maturity, the firm's asset value must be at least as high as the face value of the debt to avoid default. Our choice of an increasing default boundary is designed to capture the idea that firms' ability to operate when firm value is below the face value of the debt declines as debt gets closer to maturity. Our choice of an exponential form is for tractability and is unlikely to affect our results significantly.

recover $1-\alpha_{BC}$ of the *levered* value of the assets. The fraction of the value of the assets not recovered by the debtholders is assumed to be consumed in the bankruptcy process. The bankruptcy boundary is an exponential curve that increases at a rate g and is equal to the face value of debt at time T. Consequently, the bankruptcy boundary is described by $F_L e^{g(t-T)}$. The bankruptcy costs for the firm are the present value of the expected losses in bankruptcy, and are denoted by $BC_L(0)$. The levered firm liquidates assets at a rate of δ of the total value of the firm's assets, so that $\delta V(t)dt$ equals the sum of the after-tax coupon paid to debt holders [$(1-\tau)C_L dt$] and a time varying dividend $div(t)dt$ paid to equity holders over the time interval dt:

$$\delta V(t)dt = \left[div(t)+(1-\tau)C_L\right]dt. \tag{C.3}$$

Note that although we have assumed a constant asset payout rate of δ, the dividend payout is time-varying and will be less for lower $V(t)$. In particular, (C.3) may require cash infusions for low asset values.[3]

At time zero the manager's stake in the firm consists of N_{Man} $(< N_{NL})$ shares and N_{Calls} European call options with strike price K that expire at time T_u. For purposes of computational tractability, we assume that the firm buys the manager's calls from a third party. Hence, if the manager exercises the calls at time T_u, he buys N_{Calls} shares from the third party at a price of $N_{Calls}K$ dollars. We assume that the manager cannot sell or hedge either his shares or his options. In addition, at time zero the manager has $NFW(0)$ dollars of non-firm wealth. For simplicity, this wealth is assumed to grow at the risk-free rate. When the debt is chosen to maximize the manager's expected utility at time T_u, this

[3] Though our bankruptcy boundary is exogenous, cash infusions are not uncommon in models with an endogenous boundary (e.g., Leland, 1994). Another possible modeling approach is to allow renegotiation when the firm hits the default boundary. See Francois and Morellec (2003) and Fan and Sundaresan (1999) for models with renegotiation.

utility is described by

$$U\left(Wealth_{T_u}\right)=\frac{\left(Wealth_{T_u}\right)^{1-\gamma}-1}{1-\gamma}, \tag{C.4}$$

where γ is a risk-aversion parameter and $Wealth_{T_u}$ is the manager's total wealth at time T_u.

The value of the debt, the bankruptcy costs, and the tax shields of debt are computed from the probability density function for first hitting the exponential bankruptcy boundary. Let $f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)$ be the probability density for first hitting a boundary described by Ae^{gt} at a time t^*, where A is a constant, if the variable *V* initially has a value $V\left(0\right)>A$ and follows geometric Brownian motion with drift $r-\delta$ and volatility σ. In our model, A is the value of the bankruptcy boundary at time zero, so that A is equal to $F_L e^{-gT}$. An explicit expression for $f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)$ is provided in *Appendix C.A.*, which begins on page C-20. Next define:

$$G\left(T,V\left(0\right),A,g,r,\delta,\sigma\right)\equiv\int_0^T f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)dt^*, \tag{C.5}$$

$$H\left(T,V\left(0\right),A,g,r,\delta,\sigma\right)\equiv\int_0^T e^{-rt*} f\left(t*;V\left(0\right),A,g,r,\delta,\sigma\right)dt*, \tag{C.6}$$

and

$$I\left(T,V\left(0\right),A,g,r,\delta,\sigma\right)\equiv\int_0^T e^{-(r-g)t^*} f\left(t^*;V\left(0\right),A,g,r,\delta,\sigma\right)dt^*. \tag{C.7}$$

Closed form solutions for these expressions are derived in *Appendix C.A*. The G function is the total probability of default from time zero to T. The H function corresponds to the present value of receiving one dollar upon default, if default occurs between time zero and T. Similarly, the I function represents the present value of receiving e^{gt^*} dollars upon default, if default occurs between time zero and T.

Following Leland and Toft (1996), the value of the debt at time zero is the sum of a contribution from the coupon, a contribution from the payment to debtholders if bankruptcy occurs, and the repayment of the face value at time T if bankruptcy does not occur:

$$\begin{aligned} D_L(0) &= C_L \int_0^T e^{-rt^*}\left(1-G\left(t^*,V(0),F_L e^{-gT},g,r,\delta,\sigma\right)\right)dt^* \\ &+ \int_0^T e^{-rt^*}\left(1-\alpha_{BC}\right)\frac{TV_L(0)}{V(0)}F_L e^{-g\left(T-t^*\right)} f\left(t^*,V(0),F_L e^{-gT},g,r,\delta,\sigma\right)dt^* \\ &+ F_L\left(1-G\left(T,V(0),F_L e^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT}, \end{aligned} \qquad \text{(C.8)}$$

or

$$\begin{aligned} D_L(0) &= \frac{C_L}{r}\left(1-\left(1-G\left(T,V(0),F_L e^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT} - H\left(T,V(0),F_L e^{-gT},g,r,\delta,\sigma\right)\right) \\ &+ \left(1-\alpha_{BC}\right)\frac{TV_L(0)}{V(0)}F_L e^{-gT} I\left(T,V(0),F_L e^{-gT},g,r,\delta,\sigma\right) \\ &+ F_L\left(1-G\left(T,V(0),F_L e^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT}, \end{aligned} \qquad \text{(C.9)}$$

where

$$TV_L(0) = V(0) + TB_L(0) - BC_L(0) \qquad \text{(C.10)}$$

is the total levered value of the firm at time zero with debt. If the $TV_L(0)/V(0)$ factor were omitted from equation (C.8), then the debtholders would receive $(1-\alpha_{BC})$ of the *unlevered* value of the assets of the firm upon bankruptcy. The inclusion of this factor implements the modeling decision (reorganization) that upon bankruptcy the debtholders receive $(1-\alpha_{BC})$ of the *levered* value to a healthy firm of the remaining assets. Explicit expressions for $TB_L(0)$ and $BC_L(0)$ are provided below.

Another modeling decision involves the question of whether the firm should refinance the debt when it matures. We consider two alternative models: The first is a "static" model, in which the firm does not replace the maturing debt, and is therefore financed entirely with equity after time *T*. The second is a "dynamic" model, in which new debt is reissued when old debt matures. Since the dynamic framework seems *a priori* more appealing, and in fact Ju (1998, 2001) shows that the refinancing assumption can affect corporate financing decisions *ex ante*, we analyze the dynamic model. Nonetheless, it is convenient to present the solution of the dynamic model in terms of that for the static model that we develop now.

C.I.a. The static model

In the static model, when the firm is forced into bankruptcy at time t^*, the bankruptcy costs are $\alpha_{BC}V\left(t^*\right)$. Hence, at time zero the value of the bankruptcy costs are

$$BC_L\left(0\right)=\int_0^T \alpha_{BC}F_L e^{g\left(t^*-T\right)}e^{-rt^*} f\left(t^*;V\left(0\right),F_L e^{-gT},g,r,\delta,\sigma\right)dt^*, \qquad \text{(C.11)}$$

or

$$BC_L\left(0\right)=\alpha_{BC}F_L e^{-gT} I\left(T,V\left(0\right),F_L e^{-gT},g,r,\delta,\sigma\right). \qquad \text{(C.12)}$$

Note that we have used the *levered* value of the remaining assets to price the debt in (C.8), but the *unlevered* value of the lost assets to compute the bankruptcy costs in (C.11). We use the unlevered value of the lost assets to compute bankruptcy costs because this corresponds to the cost to the original shareholders before the firm is levered. We also computed bankruptcy costs using the levered value of the lost assets. We omit these calculations, however, because the results using this approach are virtually identical to those from the approach that uses the unlevered value.

The interest tax shields, or tax benefits of debt, accrue to the firm as long as it has not gone bankrupt. Consequently, the interest tax shields of debt in the static model can be computed by

$$TB_L\left(0\right)=\int_0^T \tau C_L e^{-rt^*}\left(1-G\left(t^*,V\left(0\right),F_L e^{-gT},g,r,\delta,\sigma\right)\right)dt^*, \qquad \text{(C.13)}$$

or

$$TB_L\left(0\right)=\frac{\tau C_L}{r}\left(1-\left(1-G\left(T,V\left(0\right),F_L e^{-gT},g,r,\delta,\sigma\right)\right)e^{-rT}-H\left(T,V\left(0\right),F_L e^{-gT},g,r,\delta,\sigma\right)\right). \quad \text{(C.14)}$$

The value of the equity is equal to the unlevered value of the assets plus the tax shields of debt minus the bankruptcy costs minus the value of the debt:

$$E_L(0) = V(0) + TB_L(0) - BC_L(0) - D_L(0). \tag{C.15}$$

In order to compute the manager's time zero expectation of his utility at time T_u, let $V^K(T_u)$ be the value of the firm's assets at time T_u that makes a share of stock worth K at time T_u. Then the manager's time zero expectation of his utility at time T_u is the sum of three components. The first component is a function of the density for the value of the firm's assets being at various levels above $V^K(T_u)$ at time T_u without having touched the bankruptcy boundary between time zero and time T_u. The second component is a function of the density for the value of the firm's assets being at various levels below $V^K(T_u)$ at time T_u without having touched the bankruptcy boundary between time zero and time T_u. The third component is the utility derived from his non-firm wealth if the bankruptcy boundary is hit. Let $\rho(V(T);V(0),T,A,g,\mu,\delta,\sigma)$ be the density function for starting at a value $V(0) > A$ and being at $V(T) > Ae^{gT}$ at time $T > 0$ without ever hitting the boundary Ae^{gt} in the interval $t \in [0,T]$ when the V process follows geometric Brownian motion with drift $\mu - \delta$ and volatility σ. An explicit expression for $\rho(V(T);V(0),T,A,g,\mu,\delta,\sigma)$ is presented in *Appendix C.A*. Then at time zero, the manager's expectation of his utility at time T_u is given by

$$
\begin{aligned}
Utility_L(0) = & \int_{V^K(T_u)}^{\infty} U\left\{NFW(T_u) + \frac{N_{Man} + N_{Calls}}{N_L}\left[V(T_u) + TB_L(T_u) - BC_L(T_u) - D_L(T_u)\right] - N_{Calls}K\right\} \\
& \times \rho\left(V(T_u); V(0), T_u, F_L e^{-gT}, g, \mu, \delta, \sigma\right) dV(T_u) \\
& + \int_{F_L e^{-g(T-T_u)}}^{V^K(T_u)} U\left\{NFW(T_u) + \frac{N_{Man}}{N_L}\left[V(T_u) + TB_L(T_u) - BC_L(T_u) - D_L(T_u)\right]\right\} \\
& \times \rho\left(V(T_u); V(0), T_u, F_L e^{-gT}, g, \mu, \delta, \sigma\right) dV(T_u) \\
& + U\left(NFW(T_u)\right) \int_0^{T_u} f\left(t; V(0), F_L e^{-gT}, g, \mu, \delta, \sigma\right) dt,
\end{aligned} \tag{C.16}
$$

where $V^K(T_u)$ satisfies the following equation:

$$
K = \frac{V^K(T_u) + TB_L(T_u) - BC_L(T_u) - D_L(T_u)}{N_L}. \tag{C.17}
$$

Note that all terms on the right hand side of equation (C.17) are a function of $V^K(T_u)$ and N_L is the number of shares after the proceeds of the debt are used to retire some of the original shares.

C.I.b. The dynamic model

Next we extend the model to a more realistic dynamic setting. If the firm has not gone bankrupt at the end of T years, the firm issues new *T*-year debt at time T. The new debt has a coupon of $C_L V(T)/V(0)$. Similarly, as shown in *Appendix C.A.*, all other securities will be scaled by a factor of $V(T)/V(0)$, because at time T the firm is identical to itself at time zero, except that it is $V(T)/V(0)$ as large. The process of issuing new *T*-year debt when the old debt matures continues indefinitely unless the firm goes bankrupt. If bankruptcy occurs at time $t^* < T$ (before debt matures), we allow the debtholders to become the new shareholders and they optimally lever the remaining assets (1-α_{BC}) $F_L e^{-g(T-t^*)}$ after the bankruptcy process consumes α_{BC} $F_L e^{-g(T-t^*)}$ of the assets $F_L e^{-g(T-t^*)}$ at the bankruptcy boundary.

In this dynamic setting, the price of the debt is still given by equation (C.9). The firm value, however, will reflect the costs and benefits of the debt issued in the future. In determining the total tax shields and total bankruptcy costs associated with current and future debt issues, the following quantity is useful:

$$\phi = \int_{F_L}^{\infty} e^{-rT} \frac{V(T)}{V(0)} \rho\left(V(T);V(0),T,A,g,r,\delta,\sigma\right) dV(T)$$
$$+ \int_0^T e^{-rt*} \frac{(1-\alpha_{BC}) F_L e^{-g(T-t*)}}{V(0)} f\left(t*;V(0),F_L e^{-gT},g,r,\delta,\sigma\right) dt*. \tag{C.18}$$

The first term accounts for the effect of rebalancing at the debt maturity date, T, if there is no default between time zero and T. The second term accounts for the leveraging of the assets that remain after bankruptcy costs if default occurs before T.[4] We show in *Appendix C.A.* that the total tax shields and the total bankruptcy costs of debt are given by

$$TB_L^{Dynamic}(0) = \frac{TB_L(0)}{1-\phi}, \tag{C.19}$$

and

$$BC_L^{Dynamic}(0) = \frac{BC_L(0)}{1-\phi}. \tag{C.20}$$

Similar to equation (C.15), the value of the equity is equal to the unlevered value of the assets plus the tax shields of debt minus the bankruptcy costs minus the value of the debt:

$$E_L^{Dynamic}(0) = V(0) + TB_L^{Dynamic}(0) - BC_L^{Dynamic}(0) - D_L(0). \tag{C.21}$$

Finally, the manager's utility in the dynamic model is given by

[4] For more details, see equation (C.A.9), in *Appendix C.A.*, and the discussion preceding it.

$$
\begin{aligned}
Utility_L^{Dynamic}(0) = &\int_{V^K(T_u)}^{\infty} U\left\{NFW(T_u) + \frac{N_{Man} + N_{Calls}}{N_L}\left[V(T_u) + TB_L^{Dynamic}(T_u) - BC_L^{Dynamic}(T_u) - D_L(T_u)\right] - N_{Calls}K\right\} \\
&\quad \times \rho\left(V(T_u); V(0), T_u, F_L e^{-gT}, g, \mu, \delta, \sigma\right) dV(T_u) \\
&+ \int_{F_L e^{-g(T-T_u)}}^{V^K(T_u)} U\left\{NFW(T_u) + \frac{N_{Man}}{N_L}\left[V(T_u) + TB_L^{Dynamic}(T_u) - BC_L^{Dynamic}(T_u) - D_L(T_u)\right]\right\} \\
&\quad \times \rho\left(V(T_u); V(0), T_u, F_L e^{-gT}, g, \mu, \delta, \sigma\right) dV(T_u) \\
&+ U\left(NFW(T_u)\right)\int_0^{T_u} f\left(t; V(0), F_L e^{-gT}, g, \mu, \delta, \sigma\right) dt.
\end{aligned} \tag{C.22}
$$

The details for computing the manager's utility in this dynamic model are provided in *Appendix C.A*.

C.II. Calibrating the model

In choosing the amount of debt that will be issued, a face value, F_L, of 10-year debt (i.e., $T = 10$ years) is chosen to maximize either the total value of the levered firm or the manager's expected utility one year in the future (i.e., $T_u = 1$). The total value of the firm's assets before the firm is levered, $V(0)$, is normalized to $100, which is divided among 100 shares, each worth $1. We assume that the manager of the firm owns 0.32 of a share of stock and a 1-year exchange traded European call option on an additional 0.38 share.[5] The strike price for the call option is set equal to the time zero value of a share of equity of the firm without debt, $1. For the base-case, the manager's non-firm wealth is assumed to equal the time-zero value of the shares that the manager owns, $0.32. Consistent with the literature, we assume the manager's risk aversion parameter γ equals 2.[6]

Given these assumptions, calibration of the model requires estimates of (1) the risk-free rate, *r*, (2) the effective tax rate, τ, (3) the volatility of the total value of the firm, σ, (4) the debtholder bankruptcy recovery rate, $(1-\alpha_{BC})$, (5) the exponent of the exponential function characterizing the

[5] The manager's stock and option holdings represent the median values for managers at 1,405 firms for which sufficient data to estimate these figures are available for 1999 in the ExecuComp database.
[6] See pages 258-260 of Ljungqvist and Sargent (2000) for a discussion and interpretation of this value.

bankruptcy boundary, g, (6) the level of dividends, *DivRate*, paid by the firm, and (7) the drift parameter for the total value of the firm, μ. We estimate these parameters using data from the end of January 2001.

As our estimate of the risk-free rate, we use the rate on 10-year Treasury bonds as of January 30, 2001, as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. This rate equals 5.22%.

To estimate the tax rate used to calculate the tax shields from the debt, we use data on estimated marginal tax rates (before interest expense) provided by John Graham, who constructed these estimates using the approach described in Graham (1996). In particular, for the base case, we assume that the tax rate equals the median marginal tax rate of 34% for the 5,519 firms for which 1999 estimates are available.

The volatility of the total value of the firm's assets, σ, the debtholder bankruptcy recovery rate, $(1-\alpha_{BC})$, and the exponent of the bankruptcy boundary function, g, are selected to yield an expected recovery in bankruptcy equal to 45% of the face value of debt and a spread over the 10-year Treasury bond rate for the firm's debt that equals 1.90%, for a firm with the median debt to total capital ratio of 22.62% among firms in the Compustat database in 2000. The 45% recovery target is broadly consistent with recovery rates published by Hamilton, Gupton, and Berhault (2001). For the 1981 to 2000 period, Hamilton, Gupton, and Berhault estimate the mean default recovery rates for senior secured bonds, senior unsecured bonds, and subordinated bonds of all ratings to equal 53.9%, 47.4%, and 32.3%, respectively. The 1.90% spread over the Treasury bond rate equals the spread for 10-year A-rated corporate debt as of January 30, 2001, as reported in the February 7, 2001 edition of Standard & Poor's *The Outlook*. The volatility of the total value of the firm's assets, σ, is estimated this way to be 38.02% (0.3802). This value implies a volatility of the value of the typical firm's equity of 48.09%. The bankruptcy recovery rate, relative to the levered value to a healthy firm of the remaining assets at the time of default, and the exponent of the bankruptcy boundary function for our base case equal 0.5090 ($\alpha_{BC} = 0.4910$) and 3.69%

($g = 0.0369$), respectively.[7]

In our calibration procedure, we do not, and cannot, distinguish between the effects of the direct and indirect costs of financial distress. The calibrated bankruptcy parameter values, $(1-\alpha_{BC})$ and g, are those that ensure that the model output matches the 1.90% spread over the Treasury bond rate and the 45% expected recovery in bankruptcy. Thus, they reflect the total direct and indirect costs of bankruptcy.

We set the dividend rate, *DivRate*, equal to 1.5% in the base case. Because this rate is stated as a percentage of the unlevered value of the firm, we use a number that is on the lower end of the 1.5% to 2.0% dividend yield paid by public firms at the beginning of 2001.

We select a value for the drift parameter of the firm, μ, by implementing an argument similar to one provided in Merton (1974). We begin by formally writing the dynamics of the equity's value as

$$dE = \left(\mu_E - \delta_E\right) E\ dt + \sigma_E E\ dZ_E. \tag{C.23}$$

By Ito's lemma and the dynamics of the firm under the physical measure given in equation (C.1), we can also write the dynamics for E as

$$dE = \left[\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 E}{\partial V^2} + \left(\mu - \delta\right) V \frac{\partial E}{\partial V} + \frac{\partial E}{\partial t}\right] dt + \sigma V \frac{\partial E}{\partial V}\ dZ. \tag{C.24}$$

Matching the coefficients on the drift components of equations (23) and (24) yields

$$\mu = \frac{\left(\mu_E - \delta_E\right) E - \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 E}{\partial V^2} - \frac{\partial E}{\partial t}}{V \frac{\partial E}{\partial V}} + \delta. \tag{C.25}$$

We set μ_E equal to 11.22% by assuming an equity risk premium of 6% over our risk free rate of 5.22%.

When the rest of the quantities on the right hand side of equation (C.25) are computed from the calibrated

[7] The asset volatility, σ, in our model is the *annual* volatility of the returns of the firm's asset value. While the model is in continuous time and the parameter values, like the interest rate, *r*, and σ, are annualized, the spread of any *finite* maturity bond over a similar Treasury bond is consistent with the use of continuous time and annualized parameter values.

values for our standardized firm with a debt to total capital ratio of 22.62%, the equation yields our base case value for μ of 10.63%.

Panel A of Table C.1 summarizes our parameter choices. These choices are used to derive the values of the variables that are presented in Panel B of Table C.1.

C.III. Optimal Capital Structure Estimates from the Manager's Perspective

Using the above parameters, we calculate the optimal capital structure from the manager's perspective by choosing the debt level that maximizes the value of his utility function rather than the total levered firm value. The results from this model are presented in Table C.2. When asset volatility is low, the manager prefers more leverage than do the shareholders (42.75% in Table C.2 vs. 40.71% in Table 2 (see main text)), indicating that the increase in the value of the manager's options associated with higher leverage outweighs the manager's dislike for risk when he chooses the firm's capital structure. On the other hand, when asset volatility is high, the value of the options is already quite high, even without any leverage. In this case, the effect of managerial risk aversion dominates the option effect and the manager chooses less leverage than the shareholders would like (6.39% in Table C.2 vs. 8.34% in Table 2).[8]

Panel B of Table C.2 reports the leverage ratios that maximize the manager's expected utility when the maturity of his options and the time at which expected utility is evaluated equals $T_u = 5$ years, rather than $T_u = 1$ year. Changing the manager's horizon from 1 to 5 years does not have a major impact on the leverage ratio that he finds optimal. For low (high) asset volatilities, the leverage ratios are higher (lower) than when $T_u = 1$. The reason is that when T_u is larger, there is more variability in the equity price at the time when the manager evaluates his utility. For low levels of asset volatility, the impact of this higher variability on the value of the manager's options outweighs his aversion toward taking risk.

[8] In an elegant unified model incorporating tax shields and default costs of debt financing, as well as managerial incentives for empire-building, Morellec (2004) has shown that managerial discretion may also explain the observed low leverage ratios. In contrast to our model that emphasizes managerial risk-aversion, Morellec focuses on agency problems associated with the tendency to empire-build.

For high levels of asset volatility, however, the effect of leverage on the value of the manager's options is relatively less important than his concerns regarding risk.

Even though there is little difference between the leverage values that maximize firm value (and hence shareholder value) in Table 2 and the leverage values that maximize manager utility in Table C.2, the fact that they are not identical indicates that there is a conflict between shareholders and managers. In this appendix we calibrate the manager's compensation package to reflect those typically observed in practice in order to investigate the size of the conflict. An interesting extension would be to study the compensation packages that are optimal from the point of view of the shareholders in the context of a model such as this one.

We also estimated the capital structure that maximizes the manager's utility for the 15 firms discussed in Section C of the main text. The value-weighted average maturity of the debt and cost of debt for each firm, estimates of each firm's marginal tax rate for 1999 (obtained from John Graham), and the actual stock and option holdings of each CEO are used in these calculations. The stock and option holdings for the CEO's are taken from the 2000 proxy statements filed by the sample firms with the SEC.

Table C.3 reports the firm-specific model inputs, estimated asset volatility and bankruptcy parameters, as well as the actual, share value-maximizing, and manager utility-maximizing debt to total capital ratios for each of the 15 firms. While most of the utility-maximizing debt-to-total capital ratios are similar to the corresponding share value-maximizing ratios, they differ substantially in cases where the debt maturity is long. This is especially apparent for Ravenswood Winery and Mead in Table C.3. In cases with long maturity debt, the preference of stockholders for more debt, illustrated in Table 2, is offset to a large extent by the managers' risk aversion.

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TABLE C.1

Model Parameters

Panel A. Chosen Parameters

Variable	Calibrated Value	Variable Description
T	10	Time at which debt matures
T_u	1	Time at which manager evaluates utility and options mature
$V(0)$	\$100	Value of assets without debt
N_{NL}	100	Total shares outstanding without debt
N_{Man}	0.32	Number of shares owned by manager
N_{Calls}	0.38	Number of exchange traded European calls owned by manager
K	\$1	Strike price of calls
$NFW(0)$	\$0.32	Manager's non-firm wealth in dollars at time zero
γ	2	Manager's risk aversion parameter
r	5.22%	Annualized risk-free rate
τ	34%	Effective tax rate for debt tax shields
σ	38.02%	Volatility of value of firm assets
α_{BC}	0.4910	Bankruptcy cost parameter (1 - debtholder bankruptcy recovery rate)
g	3.69%	Exponent of bankruptcy boundary function
DivRate	1.5%	Dividend payout rate to equity holders as a percentage of the unlevered value of the firm.
μ	10.63%	Drift of value of firm assets

TABLE C.1 continued

Panel B. Derived Parameters

Variable	Variable Description
F_L	Face value of debt
C_L	Constant annualized coupon rate paid on debt. This is set to price the debt at par.
$D_L(0)$	Initial total value of debt
N_L	Total shares outstanding with debt
$E_L(0)$	Initial total value of equity with debt
$BC_L(0)$	Initial total value of bankruptcy costs
$TB_L(0)$	Initial total value of tax shields of debt
$NFW(T_u)$	Value of manager's non-firm wealth at time T_u
$Utility(0)$	Expected future value of manager's utility without debt
$Utility_L(0)$	Expected future value of manager's utility with debt
ϕ	Scaling factor that accounts for future rebalancing for total tax shields and default costs.
$E_L^{Dynamic}(0)$	Initial total value of equity with debt
$BC_L^{Dynamic}(0)$	Initial total value of bankruptcy costs
$TB_L^{Dynamic}(0)$	Initial total value of tax shields of debt
δ	After tax cash payout rate to both debtholders and equity holders as a percentage of the unlevered value of the firm.
$V^K(T_u)$	Value of assets that makes a share of stock worth K dollars at time T_u

TABLE C.2

Model Output for Firms with Different Firm Asset Volatilities Where Objective is to Maximize the Manager's Utility

		Volatility of Firm Asset Value								
Row	Variable	13.00%	18.00%	23.00%	28.00%	33.00%	38.02%	43.00%	48.00%	53.00%
Panel A. $T_u = 1$										
1.	Debt/total capital	42.75%	34.15%	27.70%	22.18%	17.60%	13.98%	10.87%	8.33%	6.39%
	Equity:									
2.	Value of equity without debt	$100	$100	$100	$100	$100	$100	$100	$100	$100
3.	Number of shares without debt	100	100	100	100	100	100	100	100	100
4.	Value of equity with debt	$71.76	$79.39	$84.25	$88.06	$90.95	$92.97	$94.66	$95.97	$96.88
5.	Number of shares with debt	57.25	65.85	72.30	77.82	82.40	86.02	89.13	91.67	93.61
6.	Change in share price	$0.253	$0.206	$0.165	$0.132	$0.104	$0.081	$0.062	$0.047	$0.035
	Debt:									
7.	Face value of debt	$53.58	$41.17	$32.27	$25.10	$19.43	$15.11	$11.55	$8.72	$6.61
8.	Value of debt	$53.58	$41.17	$32.27	$25.10	$19.43	$15.11	$11.55	$8.72	$6.61
9.	Coupon	$2.898	$2.264	$1.815	$1.443	$1.142	$0.910	$0.711	$0.548	$0.425
10.	Bankruptcy costs	$3.253	$3.726	$4.249	$4.413	$4.349	$4.225	$3.857	$3.399	$2.983
11.	Tax benefits	$28.598	$24.285	$20.772	$17.578	$14.726	$12.305	$10.066	$8.086	$6.475
Panel B. $T_u = 5$										
12.	Debt/total capital	48.42%	35.33%	26.27%	19.53%	14.45%	10.29%	7.07%	4.61%	3.19%

The values are for a manager with a risk aversion parameter of 2 and who owns 0.32 shares and options on 0.38 shares.

TABLE C.3

Individual Firm Estimates

	Firm-Specific Model Inputs					Estimated					
	Debt Maturity (T)	Cost of Debt Less Treasury Yield	Tax Rate (τ)	Manager Holdings Shares	Manager Holdings Options	Volatility of Asset Value (σ)	Exponent of Bankruptcy Boundary Function (g)	Bankruptcy Cost Parameter (α_{BC})	Actual Debt/ Total Capital	Estimated Debt/Total Capital: Value Maximizing	Estimated Debt/Total Capital: Utility Maximizing
Panel A. Wholesale Distribution Firms											
Hughes Supply Inc.	6.5	5.75%	35.0%	2.02%	0.42%	27.67%	3.95%	0.5004	56.36%	22.24%	21.79%
Avnet, Inc.	2.0	3.00%	35.0%	0.07%	1.79%	33.52%	3.59%	0.5413	43.32%	22.90%	21.12%
Tessco Technologies, Inc.	4.0	1.35%	34.0%	19.06%	8.44%	62.51%	2.71%	0.5384	7.37%	5.74%	4.44%
Audiovox Corp.	1.5	0.50%	35.4%	18.05%	2.33%	70.60%	7.98%	0.5415	8.46%	8.69%	6.71%
Grainger (W.W.) Inc.	1.5	0.50%	35.0%	0.12%	0.44%	71.98%	4.17%	0.5455	8.00%	8.19%	6.69%
Panel B. Beer and Wine Manufacturing											
Robert Mondavi Corp.	8.0	2.25%	35.0%	10.82%	1.98%	33.69%	0.00%	0.5500	29.21%	15.07%	14.22%
Willamette Valley Vineyards	13.0	2.25%	34.0%	22.16%	2.24%	33.03%	3.96%	0.4547	32.76%	22.04%	19.48%
Pyramid Breweries	5.0	2.25%	34.2%	0.62%	3.02%	34.34%	3.75%	0.5239	31.74%	17.32%	16.58%
Golden State Vintners	3.5	2.25%	35.0%	9.80%	10.66%	33.01%	3.65%	0.5338	35.52%	19.76%	19.09%
Ravenswood Winery	20.0	1.35%	35.0%	8.77%	2.06%	46.95%	3.15%	0.4397	8.77%	25.84%	13.84%
Panel C. Paper and Allied Products Manufacturing											
Boise Cascade Corp.	9.0	3.00%	12.2%	0.02%	1.69%	27.95%	3.92%	0.4886	48.19%	15.58%	13.53%
Kimberley-Clark	10.0	1.35%	35.0%	0.07%	0.29%	53.87%	3.07%	0.5087	6.24%	7.57%	5.93%
Mead	23.0	2.25%	35.0%	0.06%	0.69%	37.68%	3.81%	0.3438	30.75%	52.55%	26.81%
P H Glatfelter Co.	3.0	2.25%	35.0%	0.31%	0.71%	32.80%	3.93%	0.5356	37.01%	20.82%	20.14%
Wausau-Mosinee Paper Corp.	5.0	2.25%	29.5%	0.03%	0.99%	35.53%	3.79%	0.5238	30.90%	15.81%	14.40%

Appendix C.A.

In this appendix we develop the dynamic model in more detail and then describe the procedure for computing the manager's utility at time T_u.

C.A.I. The dynamic model

To obtain $G(T), H(T),$ and $I(T)$[9] in (C.5), (C.6), and (C.7), we require the first passage time density function. To compute it, we first define

$$x(t) \equiv \log\left(\frac{V(t)}{F_L e^{-g(T-t)}}\right). \tag{C.A.1}$$

One application of Ito's lemma under the risk-neutral measure yields

$$dx = \left(r - \delta - g - \sigma^2/2\right)dt + \sigma\, dZ^Q(t). \tag{C.A.2}$$

Consequently, $x(t)$ is a Brownian motion with drift $m \equiv r - \delta - g - \sigma^2/2$ and diffusion σ, starting at $x_0 = \log\left(\frac{V(0)}{F_L e^{-gT}}\right)$. From Ingersoll (1987), the first-passage time density function $f(t)$ for crossing the origin is given by

$$f(t) = \frac{x_0}{\sigma\, t^{3/2}} n\left(\frac{x_0 + mt}{\sigma\, t^{1/2}}\right), \tag{C.A.3}$$

where $n(\bullet)$ is the standard normal density function. Now, lengthy, but straightforward calculations yield[10]

$$G(T) = N\left[h_1(T)\right] + \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-2a} N\left[h_2(T)\right], \tag{C.A.4}$$

$$H(T) = \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a+z} N\left[q_1(T)\right] + \left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a-z} N\left[q_2(T)\right], \tag{C.A.5}$$

[9] For simplicity, we omit the other arguments of these functions.
[10] Explicit derivation is available upon request.

$$I(T)=\left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a+\bar{z}} N\left[\bar{q}_1(T)\right]+\left(\frac{V(0)}{F_L e^{-gT}}\right)^{-a-\bar{z}} N\left[\bar{q}_2(T)\right], \qquad \text{(C.A.6)}$$

where

$$h_1(T)\equiv\left(\frac{-x_0-a\sigma^2 T}{\sigma\sqrt{T}}\right), \qquad h_2(T)\equiv\left(\frac{-x_0+a\sigma^2 T}{\sigma\sqrt{T}}\right),$$

$$q_1(T)\equiv\left(\frac{-x_0-z\sigma^2 T}{\sigma\sqrt{T}}\right), \qquad q_2(T)\equiv\left(\frac{-x_0+z\sigma^2 T}{\sigma\sqrt{T}}\right), \qquad \text{(C.A.7)}$$

$$\bar{q}_1(T)\equiv\left(\frac{-x_0-\bar{z}\sigma^2 T}{\sigma\sqrt{T}}\right), \qquad \bar{q}_2(T)\equiv\left(\frac{-x_0+\bar{z}\sigma^2 T}{\sigma\sqrt{T}}\right),$$

$$a\equiv\frac{\left(r-\delta-g-\sigma^2/2\right)}{\sigma^2}, \qquad z\equiv\frac{\left[\left(a\sigma^2\right)^2+2r\sigma^2\right]^{1/2}}{\sigma^2}, \qquad \bar{z}\equiv\frac{\left[\left(a\sigma^2\right)^2+2\left(r-g\right)\sigma^2\right]^{1/2}}{\sigma^2}.$$

In these expressions, $N(\bullet)$ is the cumulative standard normal distribution function.

Given $G(T), H(T)$, and $I(T)$, the values of the debt, bankruptcy costs, and tax shields of the current debt are given by equations (C.9), (C.12), and (C.14), respectively. The total firm value in the static model, when debt is issued only once by the firm, is given by the value of the firm's unlevered assets plus the tax shields of debt (C.14) minus the bankruptcy costs (C.12)

$$TV_L(0)=V(0)+TB_L(0)-BC_L(0)\ . \qquad \text{(C.A.8)}$$

We now turn our attention to the dynamic model. In this model the firm repeatedly and optimally issues *T*-year maturity debt until it goes bankrupt. Obviously, the optimal coupon for the new issues will depend on the firm value when the future debt is issued. We note, however, the following scaling property: If the optimal coupon of the first (initial) debt issue is C_L, then the optimal coupon in future issues will be scaled by the ratio of the asset value $V(t)$ when the new debt is issued to the initial asset value $V(0)$. The reason for this is that at time t the firm is identical to itself at time zero, except that it is $V(t)/V(0)$ as large because the asset value follows a proportional process (geometric Brownian motion). Therefore, if no bankruptcy has occurred by the time that the initial debt matures at T, the optimal coupon of the new debt will be $C_L\left(V(T)/V(0)\right)$. Now, if bankruptcy occurs at $t^* < T$, the asset value will be $F_L e^{-g\left(T-t^*\right)}$. We allow the debtholders to become the new shareholders, and they optimally lever the remaining asset value (1-α_{BC}) $F_L e^{-g\left(T-t^*\right)}$ after the bankruptcy process consumes $\alpha_{BC}\ F_L e^{-g\left(T-t^*\right)}$. Thus, the optimal coupon after bankruptcy reorganization is $C_L \dfrac{(1-\alpha_{BC})F_L e^{-g(T-t^*)}}{V(0)}$. In fact, all future issues of debt will be scaled by the ratio of the firm's asset value when the new debt is issued to its asset value when the old debt is issued.

Even though only the current issue of debt is outstanding at time zero, the tax shields and bankruptcy costs reflect all expected future debt issues. Let $TB_L^{Dynamic}(0)$ denote the total tax shields in the dynamic model. The scaling property discussed in the previous paragraph implies that the total tax shields at time T will be $TB_L^{Dynamic}(0)\left(V(T)/V(0)\right)$ if no bankruptcy has yet occurred and it will be $TB_L^{Dynamics}(0)\dfrac{(1-\alpha_{BC})F_L e^{-g(T-t^*)}}{V(0)}$ if bankruptcy occurs at *t**. Risk-neutral valuation implies that the initial *total* tax shield, $TB_L^{Dynamic}(0)$, is equal to the tax shield from the initial debt plus the conditional discounted risk-neutral expected total tax shield at time T plus the conditional discounted risk-neutral expected total tax shield if bankruptcy occurs at $t^* < T$. If we now let $TB_L(0)$ denote the tax shield from the initial issue of debt, we have[11]

$$\begin{aligned} TB_L^{Dynamic}(0) &= TB_L(0) + \int_{F_L}^{\infty} e^{-rT}\frac{V(T)}{V(0)} TB_L^{Dynamic}(0)\,\rho\left(V(T)\right)dV(T) \\ &\quad + \int_0^T e^{-rt^*}\frac{(1-\alpha)F_L e^{-g(T-t^*)}}{V(0)} TB_L^{Dynamic}(0)\, f(t^*)\,dt^* \qquad \text{(C.A.9)} \\ &= TB_L(0) + \phi TB_L^{Dynamic}(0), \end{aligned}$$

[11] For simplicity, we omit the other arguments of $\rho\left(V(T)\right)$ and $f(t^*)$.

where ϕ is defined in (C.18). Solving for $TB_L^{Dynamic}(0)$, we obtain

$$TB_L^{Dynamic}(0) = \frac{TB_L(0)}{1-\phi}. \tag{C.A.10}$$

The total tax shields have an intuitive series expansion. Each term in the expansion

$$TB_L^{Dynamic}(0) = TB_L(0)\left(1+\phi+\phi^2+\phi^3+\cdots\right) \tag{C.A.11}$$

represents the present value of the tax shields from the debt issue in each succeeding period.

To find ϕ, we require the conditional distribution of $V(T)$ such that the firm has not gone bankrupt at time T. Again, from Ingersoll (1987) we have the following conditional density function for $V(T)$:

$$\rho(V(T)) = \frac{1}{V(T)\sigma\sqrt{T}}\, n\!\left(\frac{x(T)-x_0-mT}{\sigma\sqrt{T}}\right) - \frac{e^{\frac{2mx_0}{\sigma^2}}}{V(T)\sigma\sqrt{T}}\, n\!\left(\frac{x(T)+x_0-mT}{\sigma\sqrt{T}}\right). \tag{C.A.12}$$

Using the above density function and the first passage time density $f(t^*)$ given in equation (C.A.3), tedious but straightforward derivations yield the following closed form solution for ϕ,

$$\phi = e^{-\delta T}\left(N(d_1) - \left(\frac{F_L e^{-gT}}{V(0)}\right)^{2\lambda} N(d_2)\right) + \frac{(1-\alpha_{BC})F_L e^{-gT}}{V(0)} I(T), \tag{C.A.13}$$

where $I(T)$ is given in equation (C.A.6), $\lambda = 1 + m/\sigma^2$ and

$$d_1 = \frac{-\log\left(F_L e^{-gT}/V(0)\right) + \left(r-\delta-g+\sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\log\left(F_L e^{-gT}/V(0)\right) + \left(r-\delta-g+\sigma^2/2\right)T}{\sigma\sqrt{T}}. \tag{C.A.14}$$

Similarly, the total bankruptcy costs in the dynamic model, $BC_L^{Dynamic}(0)$, are given by

$$BC_L^{Dynamic}(0) = \frac{BC_L(0)}{1-\phi}. \tag{C.A.15}$$

The total levered firm value, $TV_L^{Dynamic}(0)$, in the dynamic model equals the unlevered firm value $V(0)$, plus the total tax shields $TB_L^{Dynamic}$, less the total bankruptcy costs $BC_L^{Dynamic}(0)$,

$$TV_L^{Dynamic}(0) = V(0) + TB_L^{Dynamic}(0) - BC_L^{Dynamic}(0) = V(0) + \frac{TB_L(0) - BC_L(0)}{1-\phi}. \quad \text{(C.A.16)}$$

The optimal capital structure is obtained by maximizing either the total firm value or the manager's utility.

C.A.II. The manager's utility at time T_u

To compute the manager's utility at time T_u, we need to obtain the value of equity at T_u. We note that at T_u, part of the first period has passed. To determine the value of $E_L^{Dynamic}(T_u)$ at T_u, we first note that the total firm value at T_u, $TV_L^{Dynamic}(T_u)$, is the sum of the value of the equity and the debt at time T_u:

$$TV_L^{Dynamic}(T_u) = E_L^{Dynamic}(T_u) + D_L(T_u). \quad \text{(C.A.17)}$$

The debt $D_L(T_u)$ can be computed from equation (C.9) since its coupon (C_L), face value (F_L) and remaining maturity $(T - T_u)$, are all known. More specifically, $D_L(T_u)$ is obtained from equation (C.9) by making the following replacement: $T \to T - T_u$. Since $E_L^{Dynamic}(T_u)$ can be obtained from equation (C.A.17) if $TV_L^{Dynamic}(T_u)$ and $D_L(T_u)$ are known, we must determine $TV_L^{Dynamic}(T_u)$.

To compute $TV_L^{Dynamic}(T_u)$, we use the present value of the total cashflow $(\delta V(t) + \tau C_S)$ from T_u to T and the present value at T_u of the total firm value $TV_L^{Dynamic}(T)$ at T. From the scaling property, we have

$$TV_L^{Dynamic}(T) = TV_L^{Dynamic}(0)\frac{V(T)}{V(0)}. \quad \text{(C.A.18)}$$

Now the present value of $TV_L^{Dynamic}(T)$ is:

$$PV\left(TV_L^{Dynamic}(T)\right) = e^{-r(T-T_u)}E_{T_u}\left[TV_L^{Dynamic}(0)\frac{V(T)}{V(0)}\middle|NBC\right]$$
$$= e^{-r(T-T_u)}TV_L^{Dynamic}(0)\frac{V(T_u)}{V(0)}E_{T_u}\left[\frac{V(T)}{V(T_u)}\middle|NBC\right] \quad \text{(C.A.19)}$$
$$= TV_L^{Dynamic}(0)\frac{V(T_u)}{V(0)}\phi(T_u),$$

where 'NBC' denotes that no bankruptcy has occurred and ϕ is the factor from equation (C.A.13) that is given by

$$\phi(T_u) = e^{-\delta(T-T_u)}\left(N\left(d_1(T_u)\right) - \left(\frac{F_L e^{-g(T-T_u)}}{V(T_u)}\right)^{2\lambda} N\left(d_2(T_u)\right)\right), \quad \text{(C.A.20)}$$

where

$$d_1(T_u) = \frac{\log\left(\frac{F_L e^{-g(T-T_u)}}{V(T_u)}\right) + \left(r - \delta - g + \sigma^2/2\right)(T - T_u)}{\sigma\sqrt{T - T_u}},$$
$$\text{(C.A.21)}$$
$$d_2(T_u) = \frac{-\log\left(\frac{F_L e^{-g(T-T_u)}}{V(T_u)}\right) + \left(r - \delta - g + \sigma^2/2\right)(T - T_u)}{\sigma\sqrt{T - T_u}}.$$

Next we obtain the present value at T_u of the cashflow $(\delta V(t) + \tau C_L)$ from T_u to T. Given the value $V(T_u)$, which is random at time 0,

$$PV\left(\delta V(t) + \tau C_L\right) = \int_{T_u}^{T} e^{-r(t-T_u)} \int_{F_L e^{-g(T-t)}}^{\infty} \left[\delta V(t) + \tau C_s\right]\rho\left(V(t)\right)d\left(V(t)\right)dt$$
$$\text{(C.A.22)}$$
$$= \int_{0}^{T-T_u} e^{-rt*} \int_{F_L e^{-g(T-T_u-t*)}}^{\infty} \left[\delta V(T_u + t*) + \tau C_L\right]\rho\left(V(T_u + t*)\right)dV(T_u + t*)dt*,$$

where $t^* = T_u + t$. The inner integral is similar to the ϕ factor in equation (C.A.13). It follows that

$$PV\left(\delta V(t)+\tau C_L\right)=\int_0^{T-T_u} e^{-rt^*}\left[\begin{array}{l}\delta V\left(T_u+t^*\right)e^{(r-\delta)t^*}\left(N\left(d_1^*\right)-\left(\frac{F_L e^{-g(T-T_u)}}{V\left(T_u\right)}\right)^{2\lambda} N\left(d_2\right)\right)+ \\ \tau C_L\left(N\left(\frac{x_0^*+mt^*}{\sigma\sqrt{t^*}}\right)-e^{-\frac{2mx_0^*}{\sigma^2}} N\left(\frac{-x_0^*+mt^*}{\sigma\sqrt{t^*}}\right)\right)\end{array}\right]dt^*, \quad \text{(C.A.23)}$$

where

$$x_0^*=\log\left(\frac{V\left(T_u\right)}{F_L e^{-g(T-T_u)}}\right),$$

$$d_1^*=\frac{x_0^*+\left(r-\delta-g+\sigma^2/2\right)t^*}{\sigma\sqrt{t^*}}, \quad \text{(C.A.24)}$$

$$d_2^*=\frac{-x_0^*+\left(r-\delta-g+\sigma^2/2\right)}{\sigma\sqrt{t^*}}.$$

Although the derivation is tedious, the time integrals can be done in closed form yielding

$$PV\left(\delta V(t)+\tau C_L\right)=V\left(T_u\right)\left(\psi_1^*-\left(\frac{F_L e^{-g(T-T_u)}}{V\left(T_u\right)}\right)^{2\lambda}\psi_2^*\right)+\frac{\tau C_L}{r}\left(\psi_3^*-e^{-\frac{2mx_0^*}{\sigma^2}}\psi_4^*\right), \quad \text{(C.A.25)}$$

where

$$\begin{aligned}
\psi_1^* &= \psi\left(x_1,x_2,x_3;T-T_u\right): x_1=-\delta, x_2=\frac{r-\delta-g+\sigma^2/2}{\sigma^2}, x_3=\frac{x_0^*}{\sigma^2}>0,\\
\psi_2^* &= \psi\left(x_1,x_2,x_3;T-T_u\right): x_1=-\delta, x_2=\frac{r-\delta-g+\sigma^2/2}{\sigma^2}, x_3=-\frac{x_0^*}{\sigma^2}<0,\\
\psi_3^* &= \psi\left(x_1,x_2,x_3;T-T_u\right): x_1=-r, x_2=\frac{r-\delta-g-\sigma^2/2}{\sigma^2}, x_3=\frac{x_0^*}{\sigma^2}>0,\\
\psi_4^* &= \psi\left(x_1,x_2,x_3;T-T_u\right): x_1=-r, x_2=\frac{r-\delta-g-\sigma^2/2}{\sigma^2}, x_3=-\frac{x_0^*}{\sigma^2}<0,
\end{aligned} \quad \text{(C.A.26)}$$

and the $\psi\left(x_1,x_2,x_3;t\right)$ function is defined as,

$$\begin{aligned}
&\psi\left(x_1,x_2,x_3;t\right)=e^{x_1 t}N\left(x_2\sqrt{t}+x_3/\sqrt{t}\right)-1\left(x_3>0\right)\\
&-\frac{1}{2}\left(\frac{x_2}{x_4}+1\right)e^{x_3(x_4-x_2)}\left(N\left(x_4\sqrt{t}+x_3/\sqrt{t}\right)-1\left(x_3>0\right)\right)\\
&-\frac{1}{2}\left(\frac{x_2}{x_4}-1\right)e^{-x_3(x_4+x_2)}\left(N\left(x_4\sqrt{t}-x_3/\sqrt{t}\right)-1\left(x_3<0\right)\right).
\end{aligned} \quad \text{(C.A.27)}$$

$N(\bullet)$ is the normal distribution function, $1(\bullet)$ is the indicator function, and $x_4=\sqrt{x_2^2-2x_1}$.