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Internet Appendix for

A new method to estimate risk and return of non-traded assets from cash flows: The case of private equity funds

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Internet Appendix 1: Proof of Theorem I

The derivations are done with a one-factor market model for simplicity. A generalization to multi-factor pricing models is trivial as long as the factors are traded assets in order to measure abnormal performance with α.

Each FoF i invests an amount T_{ij} in project j at date t_{ij}. There is a total of n_i projects per FoF i and a liquidation dividend D_{ij} is paid at date d_{ij} for each project. The first project of FoF i starts at the inception date t_{0i} and the last project is terminated at the liquidation date L_i. From assumption 1, the dividend of the project j at date d_{ij} is given by

$$D_{ij} = T_{ij} \prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t}) \tag{1}$$

We then divide by $\prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t})$ and take expectations on both sides of equation (1). From assumption 1, expectations of the cross-products of the form $\varepsilon_{ij,t}\varepsilon_{ij,s}$ are equal to zero (as well as higher-order cross-products) for $t \neq s$, and we obtain

$$E_{t_{ij}} \left[\frac{D_{ij}}{\prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t})} - T_{ij} \right] = 0 \tag{2}$$

This equation simply says that the expected pricing error of a project equals zero if one uses the correct pricing model, conditional upon the information set at time t_{ij}. It is essential to note that in equation (2) we take the expectation with respect to the idiosyncratic terms $\varepsilon_{ij,t}$ present in D_{ij} and take the realized market returns $r_{m,t}$ as given. In other words, in (2) we condition upon the realized market returns. We do this because β is identified by observing a cross-section of FoFs that are exposed to different realizations of market returns.

We have also derived that because of assumption 2, equation (2) holds irrespective of the timing of the dividend d_{ij} (endogeneity). Hence, our method allows for the possibility of exit timing (e.g. early exit if good performance). The mathematical proof is available on request.

Equation (2) cannot be applied directly given the nature of our sample because we only observe the time series of cash flows at the fund level. We do observe investment and dividend cash flows separately, but we do not know which dividend corresponds to which investment and one investment can deliver multiple dividends. Our solution is to discount all cash flows from time t_{ij} to the inception date of FoF i (t_{0i}). We thus divide equation (2) by $\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})$ to obtain

$$E\left[\frac{1}{\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}\left(\frac{D_{ij}}{\prod_{t=t_{ij}+1}^{d_{ij}}(1+r_{f,t}+\alpha+\beta r_{m,t})}-T_{ij}\right)\right]=0 \qquad (3)$$

Note that the weighting factor $1/\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})$ enters the expectation as a result of assumption 1: if the pricing model is correctly specified then, by definition, pricing errors are not predictable by any variable in the information set at time t_{ij}.

Importantly, multiplying the pricing error by this weighting factor does not imply any assumption on how the investor reinvests dividends or how the investor invests his cash before it is taken by the fund at t_{ij}. The weighting factor simply weights the pricing error. We could have used any other weighting factor (observable at date t_{ij}) but our choice ensures that we only need fund-level data to estimate the parameters. This can be seen when we rewrite Equation (3) as

$$E\left[\frac{D_{ij}}{\prod_{t=t_{0i}+1}^{d_{ij}}(1+r_{f,t}+\alpha+\beta r_{m,t})}\right]=E\left[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}\right] \tag{4}$$

The moment condition (4) is the basis of the GMM estimation. We construct the sample equivalent of the expectations in equation (4) by averaging across projects within a FoF. The left hand side of (4) is estimated by

$$\overline{PV}^{D_i}(\alpha,\beta)=\frac{1}{n_i}\sum_{j=1}^{n_i}\left[\frac{D_{ij}}{\prod_{t=t_{0i}+1}^{d_{ij}}(1+r_{f,t}+\alpha+\beta r_{m,t})}\right] \tag{5}$$

which is simply the present value of all dividends of the FoF. The right hand side of (4) is then estimated by

$$\overline{PV}^{T_i}(\alpha,\beta)=\frac{1}{n_i}\sum_{j=1}^{n_i}\left[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}\right] \tag{6}$$

Note that the expressions (5) and (6) can be calculated even if we do not know the correspondence between each investment and its dividend. We then have N moment conditions, one for each FoF, so that we can construct a GMM estimator if $N \geq 2$ (in case of the market model). The first-step GMM estimator with identity weighting matrix is then the solution of

$$\min_{\alpha,\beta} \sum_{i=1}^{N} [\overline{PV}^{D_i}(\alpha,\beta) - \overline{PV}^{T_i}(\alpha,\beta)]^2 \tag{7}$$

which can directly be rewritten as the optimization in (3). As n_i tends to infinity, the averages $\overline{PV}^{D_i}$ and $\overline{PV}^{T_i}$converge to the expectations in equation (4). Therefore, the parameters estimated from the GMM optimization (7) are consistent under standard GMM regularity conditions.

Internet Appendix 2: Calibration of the shifted lognormal distribution

This appendix describes how we calibrate the parameters of the shifted lognormal distribution for the market return and idiosyncratic shocks. The shifted lognormal distribution of $e^x - c$, where $x \sim N(\mu, \sigma)$, has 3 parameters: μ, σ, and c.

For the market return, we set the minimum return c to -20% (the minimum is -20.9% in our sample). μ_m and σ_m are so that we match the average S&P 500 return and volatility over the 1980-2003 sample period. That is, μ_m and σ_m solve

$$\begin{aligned} E(R_m) &= e^{\mu_m + \frac{\sigma_m^2}{2}} - c_m \\ Var(R_m) &= (e^{\sigma_m^2} - 1)(e^{2\mu_m + \sigma_m^2}) \end{aligned} \tag{8}$$

For the idiosyncratic error $\varepsilon_{ij,t}$, c_ε is so that return is always above -100%:

$$\alpha + R_f + \beta * (c_m - R_f) + c_\varepsilon = -1 \tag{9}$$

μ_ε and σ_ε solve a system of two equations. First, like above, we have $E(\varepsilon_{ij,t}) = e^{\mu_\varepsilon + \frac{\sigma_\varepsilon^2}{2}} - c_\varepsilon = 0$. Second, we match the Cochrane (2005b) estimate for the standard deviation of idiosyncratic shocks (86% per year in a log-CAPM setting).[1]

Internet Appendix 3: Characteristics in alpha and beta

In this appendix we analyze whether the alpha and beta depend on fund charac-

[1] Cochrane (2005b) uses the following log-CAPM specification $ln(\frac{V_{t+1}}{V_t}) = R_f + a + b(R_m - R_f) + \eta$, where η is normal with variance σ^2. Doing a Taylor-expansion, we get $\frac{V_{t+1}}{V_t} = e^{R_f + a + b(R_m - R_f) + \eta} \cong (1 + R_f + a + b(R_m - R_f))e^{\eta - \sigma^2/2}$. Here we centralize η such that $E(e^{\eta - \sigma^2/2}) = 1$, and choose $\alpha = a + \sigma^2/2$ and $\beta = b$. For the CAPM we have $\varepsilon = \frac{V_{t+1}}{V_t} - (1 + R_f + \alpha + \beta(R_m - R_f))$. This gives $\varepsilon = \frac{V_{t+1}}{V_t} - (1 + R_f + \alpha + \beta(R_m - R_f)) = (1 + R_f + \alpha + \beta(R_m - R_f))(e^{\eta - \sigma^2/2} - 1)$.

Using the law of total variance and $E(\varepsilon | R_m) = 0$ (since $E(e^{\eta - \sigma^2/2}) = 1$), we have $Var(\varepsilon) = E\left[Var(\varepsilon | R_m)\right] = E\left[(1 + R_f + \alpha + \beta(R_m - R_f))^2 Var(e^{\eta - \sigma^2/2}) | R_m)\right] = E\left[(1 + R_f + \alpha + \beta(R_m - R_f))^2\right](e^{\sigma^2} - 1)e^{\sigma^2}e^{-\sigma^2}$. Hence, μ_ε and σ_ε are so that $Var(\varepsilon) = (e^{\sigma_\varepsilon^2} - 1)(e^{2\mu_\varepsilon + \sigma_\varepsilon^2}) = E\left[(1 + R_f + \alpha + \beta(R_m - R_f))^2\right](e^{\sigma^2} - 1)$, where σ is set to 0.86.

teristics. For example, Kaplan and Schoar (2005) find that fund returns (measured by Public Market Equivalent or IRR) are positively related to the fund size. Our framework allows us to investigate whether this effect is due to higher abnormal performance or higher risk exposures. We make alpha and beta a function of fund size using a dummy which equals one if committed capital is above the median value across funds. Next, we form size-sorted portfolios (i.e. FoFs) for each vintage year. This allows us to pin down the effect of size from the cross-section of moment conditions. If we would use the 14 vintage-year portfolios, size effects would only be identified to the extent that funds with different vintage years have different size. We thus form 2 portfolios per vintage year - one with large funds and one with small funds.

We show results in Table A.1. We first include the size dummy in the alpha specification only and confirm that the performance is positively related to size, but it is only significant for venture capital funds. However, when we allow beta to depend on size as well, this size effect in alpha (for venture capital funds) becomes smaller and insignificant while the size effect in beta is positive but also insignificant. Hence, once we control for differences in beta, we do not find that large funds outperform small funds. The result is similar for buyout funds.

We repeat the same exercise with a dummy capturing whether a fund is a first-time fund or not (capturing experience of the firm) and fund focus (US / Europe). Overall, we do not find significant effects for these variables, although US funds seem to have slightly better performance than European funds.

Table A.1: Risk, Abnormal Performance and Fund Characteristics

This table shows monthly abnormal performance (Alpha; in percentage) and risk loadings using a one-factor market model. Alpha and beta are specified as a function of fund characteristics as follows: Alpha = a_0 + $a_{fund\_characteristic}$*fund_characteristic and Beta = b_0 + $b_{fund\_characteristic}$*fund_characteristic. Fund characteristics include a dummy variable that is one if the fund's committed capital is larger than the median, a dummy variable that is one if the fund is not a first-time fund, and a dummy variable that is one if the fund is US focused. Estimation is executed by GMM with joint estimation of final market values and equally weighting moment conditions. Standard errors are obtained by bootstrapping and are shown between parentheses. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

	Venture Capital Funds						Buyout Funds					
Alpha (%, monthly)												
a_0	***-1.69	***-1.26	***-1.96	***-1.48	***-1.22	***-1.68	**-0.83	*-0.64	**-0.78	-0.47	**-0.77	**-0.82
	(0.12)	(0.14)	(0.19)	(0.24)	(0.43)	(0.33)	(0.34)	(0.35)	(0.34)	(0.29)	(0.35)	(0.42)
a_{size}	***0.52			0.30			0.21			-0.23		
	(0.12)			(0.27)			(0.24)			(0.38)		
$a_{experience}$		0.06			0.01			0.19			0.47	
		(0.16)			(0.47)			(0.38)			(0.81)	
$a_{US\ focused}$			***0.82			0.55			0.25			0.23
			(0.22)			(0.34)			(0.38)			(0.68)
(market) Beta												
b_0	***3.05	***2.94	***3.43	***2.53	**2.50	***2.44	***1.62	***1.43	**1.50	***1.15	***1.71	**1.52
	(0.42)	(0.48)	(0.29)	(0.67)	(1.12)	(0.64)	(0.53)	(0.41)	(0.63)	(0.35)	(0.60)	(0.59)
b_{size}				0.56						0.56		
				(0.80)						(0.44)		
$b_{experience}$					0.47						-0.51	
					(1.24)						(1.00)	
$b_{US\ focused}$						1.03						0.06
						(0.70)						(0.87)