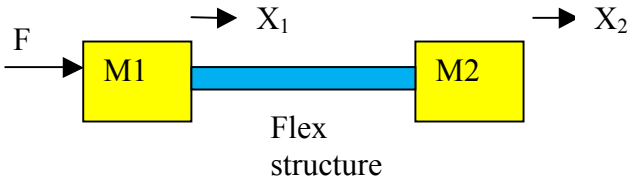
Feedforward Control

Short Course
HW 1

Consider the following model of two-masses connected by a flexible structure. One approach to model the flexible structure is to consider it as a distributed-mass system, e.g., using Finite Element Method (FEM).



The simplest model is the following

$$[M_{lumped} + M_{rod}]\ddot{U} + [C_{rod}]\dot{U} + [K_{rod}]U = \begin{bmatrix}1\\0\end{bmatrix} F$$

Where $U = \begin{bmatrix}X_1\\X_2\end{bmatrix}$ is a vector of the displacement of the two masses,

$$[M_{lumped}] = \begin{bmatrix}M_1 & 0\\0 & M_2\end{bmatrix};\quad [M_{rod}] = \frac{A_r L_r \rho_r}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix};$$

$$[K_{rod}] = \frac{A_r E_r}{L_r}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix};\ \text{and}\ [C_{rod}] = \alpha[K_{rod}]$$

ρ_r is the density of the rod, E_r is the elastic modulus of the rod, A_r is the cross-sectional area of the rod, and L_r is the length of the rod.
In the following, let

$$\frac{A_r L_r \rho_r}{6} = 1,\quad \frac{A_r E_r}{L_r} = 1,\quad M_1 = M_2 = 10, \text{ and } \alpha = 0.1$$

Let the output be the displacement of the mass further from where the force is applied, i.e., the input u is force F and the output y is displacement X_2.

1) Obtain a state space model with one FEM element for the rod. Use MATLAB to find the poles and zeros of the system when the output is X_2. (How does it differ from the case when the output is X_1?)

2) In the following let the output be X_2 Use MATLAB to find the transfer function of the system (ss2tf) and verify the poles and zeros of the TF.

3) The desired output acceleration d^2y/dt^2 is of the form Asin(wt). Choose A such that the output reaches 1 (y=1) after one period T = 10 s. The acceleration remains zero after the time period T. Obtain the velocity profile of the output as well as the position profile. You can do this analytically. But check using MATLAB integration.

4) Design your favorite feedback system. If you are having trouble consider a PD controller, but feel free to choose any controller you like. The goal is to achieve precision output tracking.

5) Simulate the output tracking of the system with this feedback (LSIM in MATLAB). What happens to the tracking when T is varied?