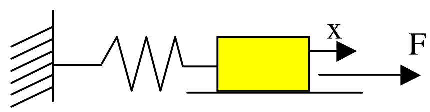
Feedforward Control
HW 1

Explicitly show all the steps.

Problem 1: Integrating Feedforward with Feedback

Consider a mass (M) connected with a spring (K) and an applied force (F). The system equations are from Newton's law



$$M\left[\frac{d^2}{dt^2}x(t)\right] + K[x(t)] = F(t)$$

where the zero position (x=0) corresponds to the un-deformed spring. Let the desired acceleration a_d have the form

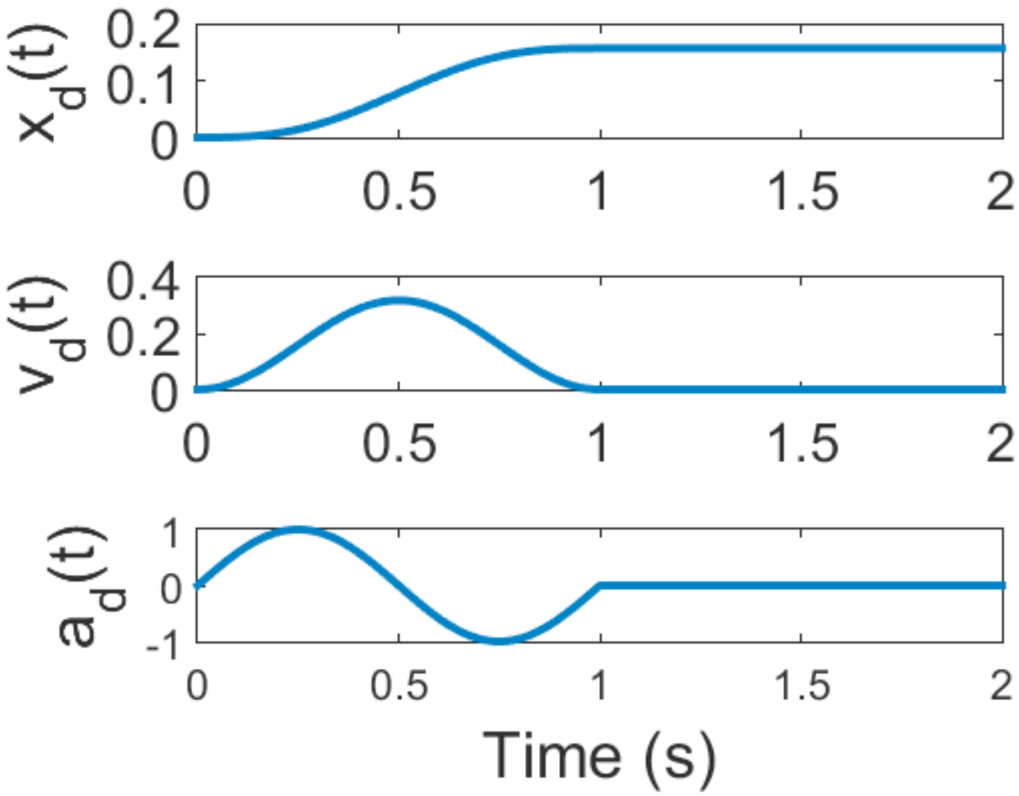
$$a_d(t) = A\sin(2\pi t/T) \;\; if \;\; 0 \le t \le T$$

and zero otherwise. For the simulations, let M=1 and K=1.

- Find an expression for the desired position x_d.
- Plot the desired acceleration, velocity, and position using MATLAB, for T=1, A=1 for the time interval [0, 2T]. Use these values for the rest of the HW.
- Find the feedforward input for this desired output
- For simplicity, consider a PD controller where the proportional gain is 0.1 and the derivative gain is 0.1.
- Simulate the response of the system with feedback alone
- Simulate the response of the system with feedback and feedforward.
- Compare the tracking results for both cases
- For the same displacement range, consider the effect of reducing the transition time by half.

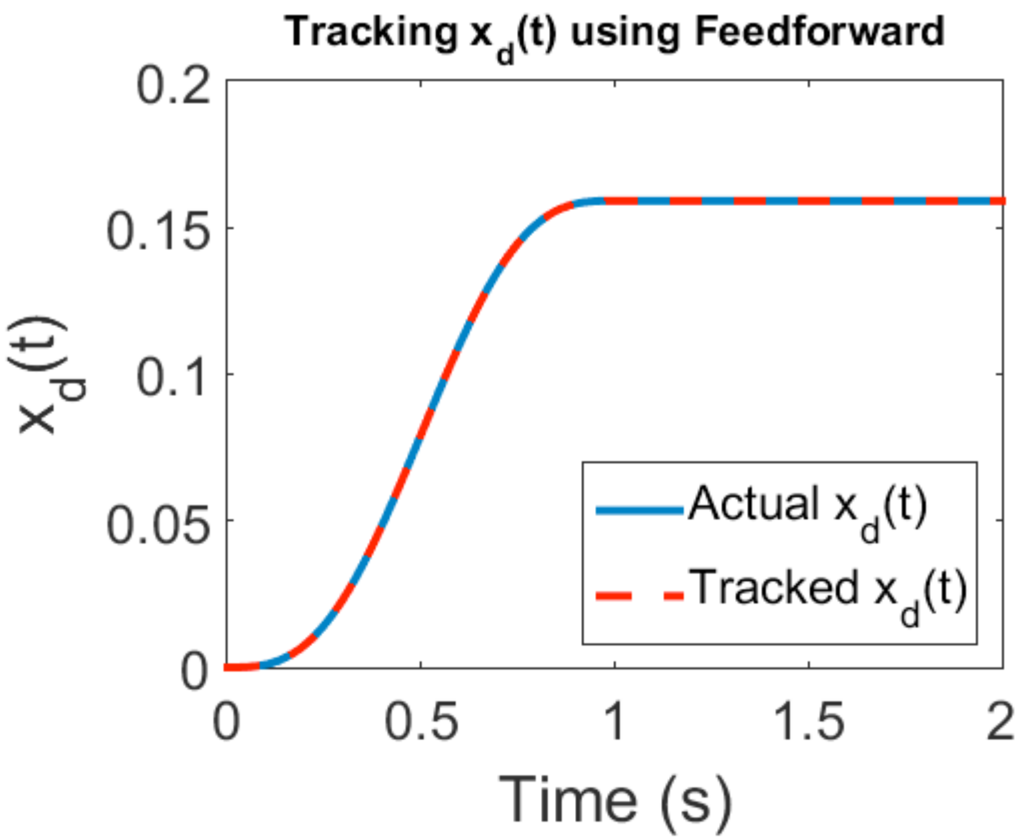
HW1 Feedforward Control


```
clear all
clc
t=0:0.001:2;
A=1;
T=1;
for i=1:length(t)
    if t(i)<=1;
        xd(i)=((A*T)/(2*pi))*[t(i)-(T/(2*pi))*sin(2*pi*t(i)/T)];
        vd(i)=((A*T)/(2*pi))*[1-cos(2*pi*t(i)/T)];
        ad(i)=A*sin(2*pi*t(i)/T);
    else
        xd(i)=xd(i-1)+0;
        vd(i)=vd(i-1)+0;
        ad(i)=ad(i-1)+0;
    end
end
figure(1)
subplot(3,1,1)
plot(t,xd,'LineWidth',3)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
subplot(3,1,2)
plot(t,vd,'LineWidth',3)
ylabel('v_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
subplot(3,1,3)
plot(t,ad,'LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('a_d(t)','Fontsize',24)
set(gca,'Fontsize',16)
```



Feedforward Input

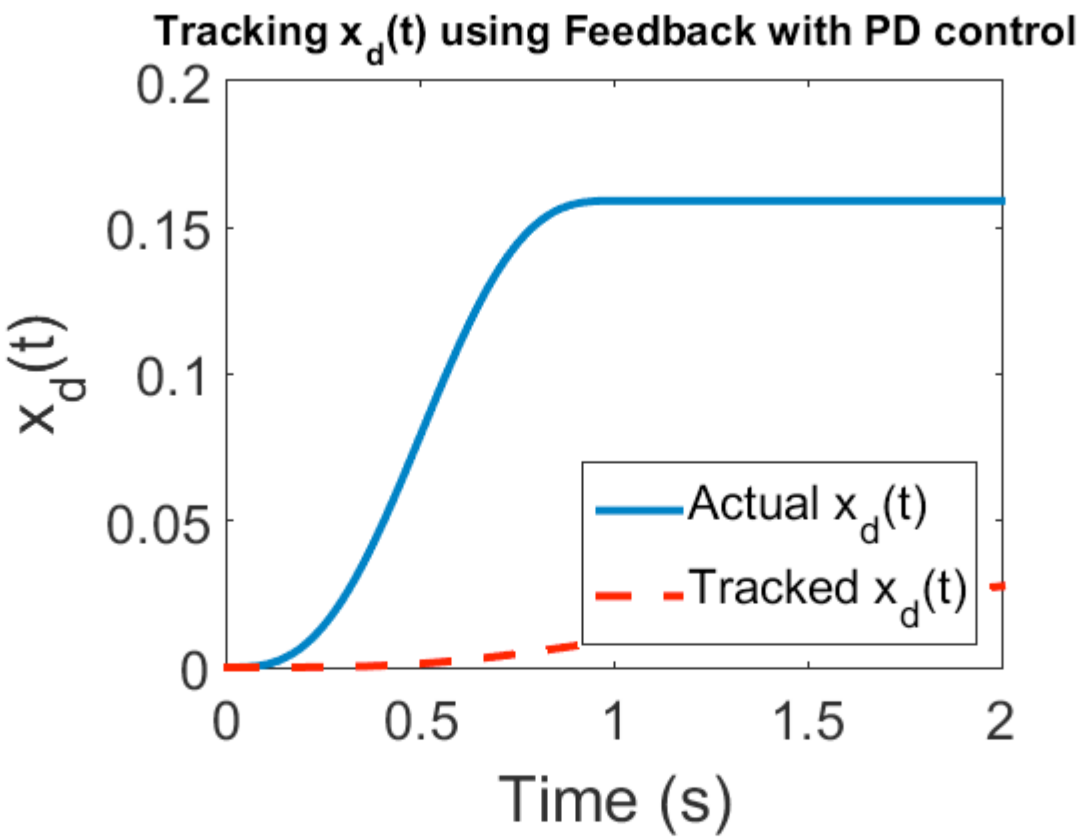
```
M=1;
k=1;
u_ff=M*ad+k*xd; %%from system equations

A=[0 1 ; -k/M 0];
B=[0; 1/M];
C=[1 0];
D=0;
sys=ss(A,B,C,D);
% x0=[0.01 0.01];
y=lsim(sys,u_ff,t);
figure
plot(t,xd,'LineWidth',3)
hold on
plot(t,y,'--r','LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
legend('Actual x_d(t)','Tracked x_d(t)','Location','southeast')
title('Tracking x_d(t) using Feedforward','Fontsize',16)
```



Simulate the response of the system with feedback

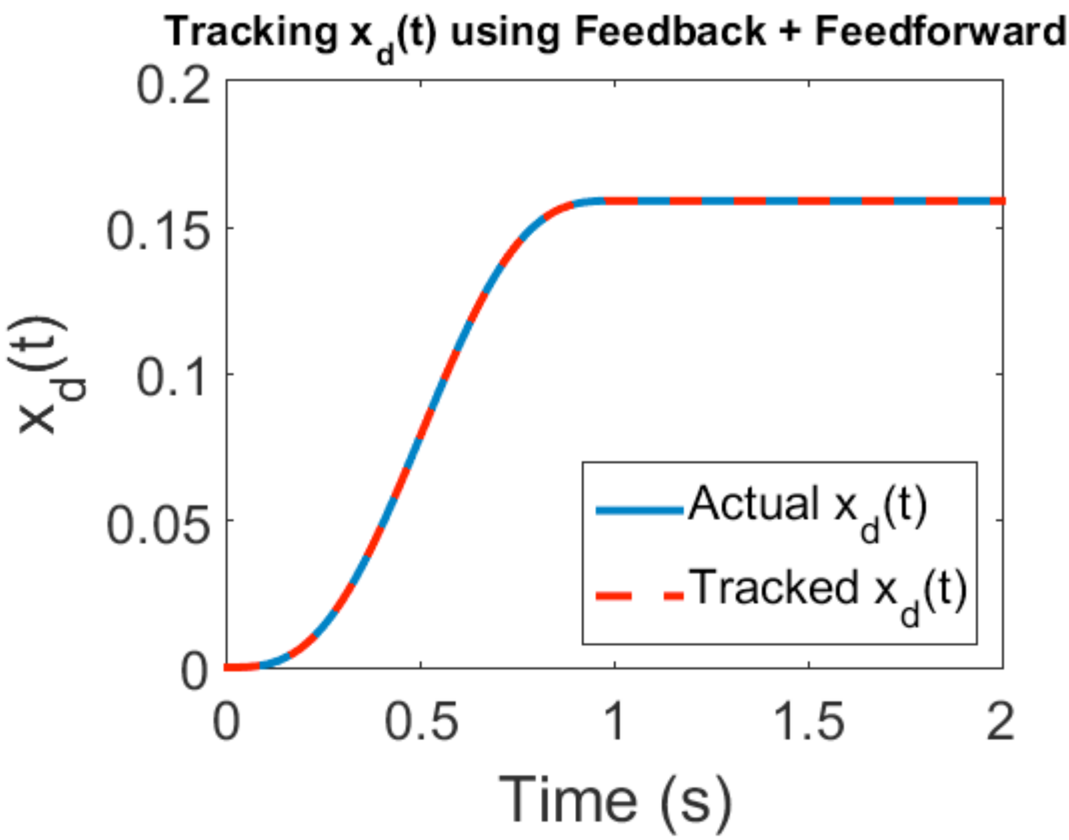
u_fb=(-0.1*xd)-(0.1*vd)%%%u=-k_fb*x

```
k_cl=[0.1 0.1];
%  k_cl=place(A,B,[-35 -45]);
u_fb=(k_cl(1)*xd)+(k_cl(2)*vd);
A=[0 1 ; -k/M 0];
B=[0; 1/M];
C=[1 0];
D=0;
sys_cl=ss(A-B*k_cl,B,C,D);
y_cl=lsim(sys_cl,u_fb,t);
figure
plot(t,xd,'LineWidth',3)
hold on
plot(t,y_cl,'--r','LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
legend('Actual x_d(t)','Tracked x_d(t)','Location','southeast')
title('Tracking x_d(t) using Feedback with PD control','Fontsize',16)
```



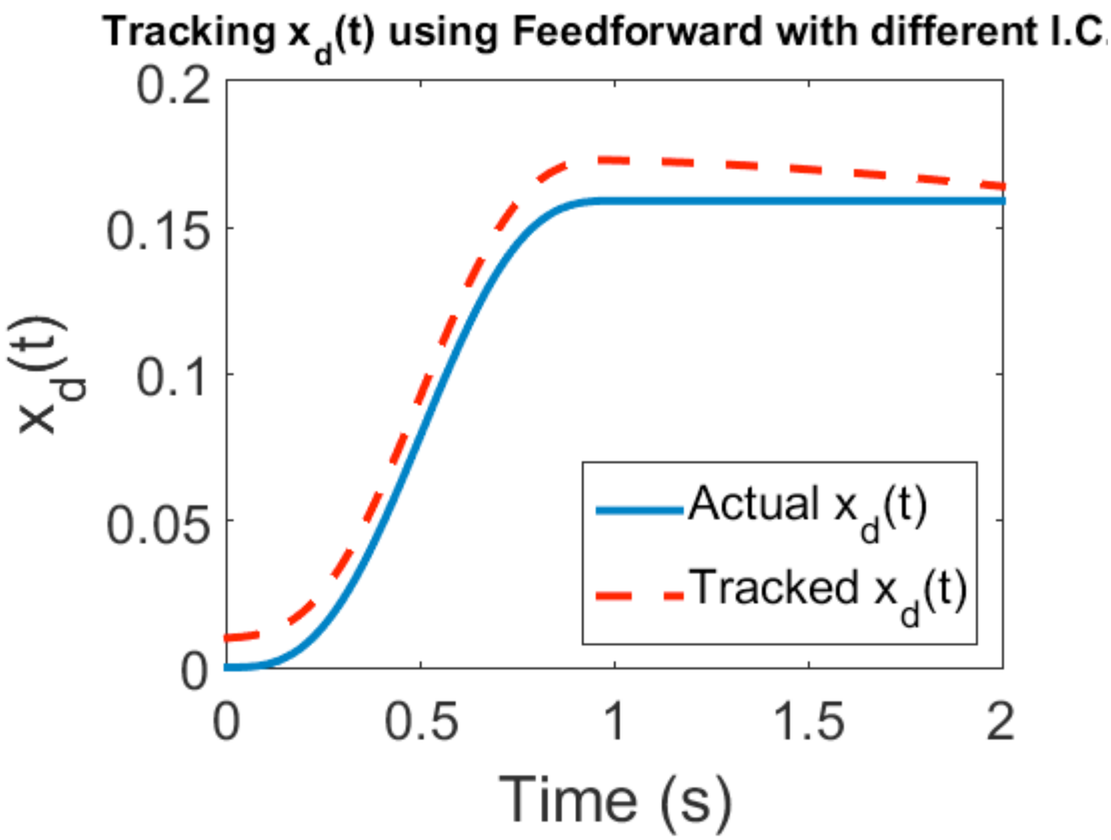
Feedback + Feedforward

```
u=u_fb+u_ff;
A=[0 1 ; -k/M 0];
B=[0; 1/M];
C=[1 0];
D=0;
sys_cl=ss(A-(B*k_cl),B,C,D);
y_cl=lsim(sys_cl,u,t);
figure
plot(t,xd,'LineWidth',3)
hold on
plot(t,y_cl,'--r','LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
legend('Actual x_d(t)','Tracked x_d(t)','Location','southeast')
title('Tracking x_d(t) using Feedback + Feedforward','Fontsize',16)
% %% Invesrion Based Approach
% A_inv=A-(B*inv(C*A*B)*C*A.^2);
% B_inv=B*inv(C*A*B);
% C_inv=-inv(C*A*B)*C*A.^2;
% D_inv=inv(C*A*B);
% sys_inv=ss(A_inv,B_inv,C_inv,D_inv);
%
% u_ff_inv=lsim(sys_inv,10^6*diff(xd,2),t(1,1:end-2));
% figure
% plot(t(1,1:end-2),u_ff_inv)
% hold on
%  plot(t,u_ff,'--r')
```



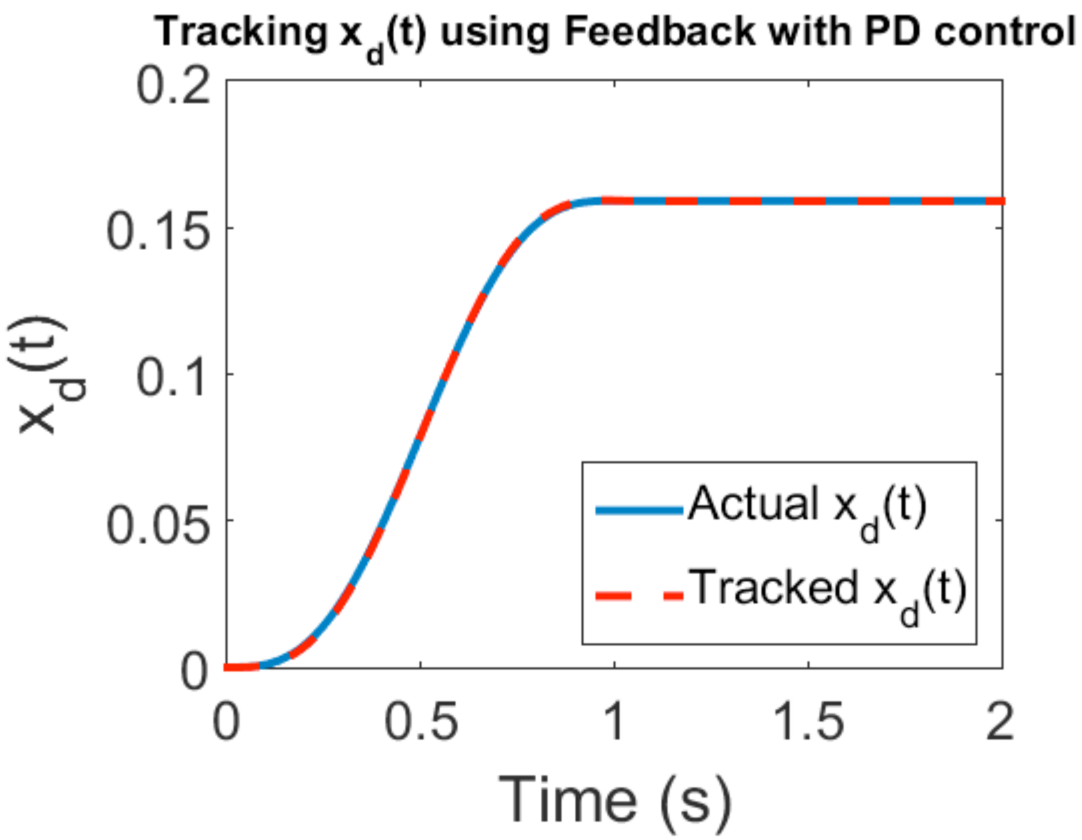
Feedforward with different intial condition

```
M=1;
k=1;
u_ff=M*ad+k*xd; %%from system equations
A=[0 1 ; -k/M 0];
B=[0; 1/M];
C=[1 0];
D=0;
sys=ss(A,B,C,D);
 x0=[0.01 0.01];
y=lsim(sys,u_ff,t,x0);
figure
plot(t,xd,'LineWidth',3)
hold on
plot(t,y,'--r','LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
legend('Actual x_d(t)','Tracked x_d(t)','Location','southeast')
title('Tracking x_d(t) using Feedforward with different I.C. ','Fontsize',16)
```



%% Simulate the response of the system with feedback with different controller gain

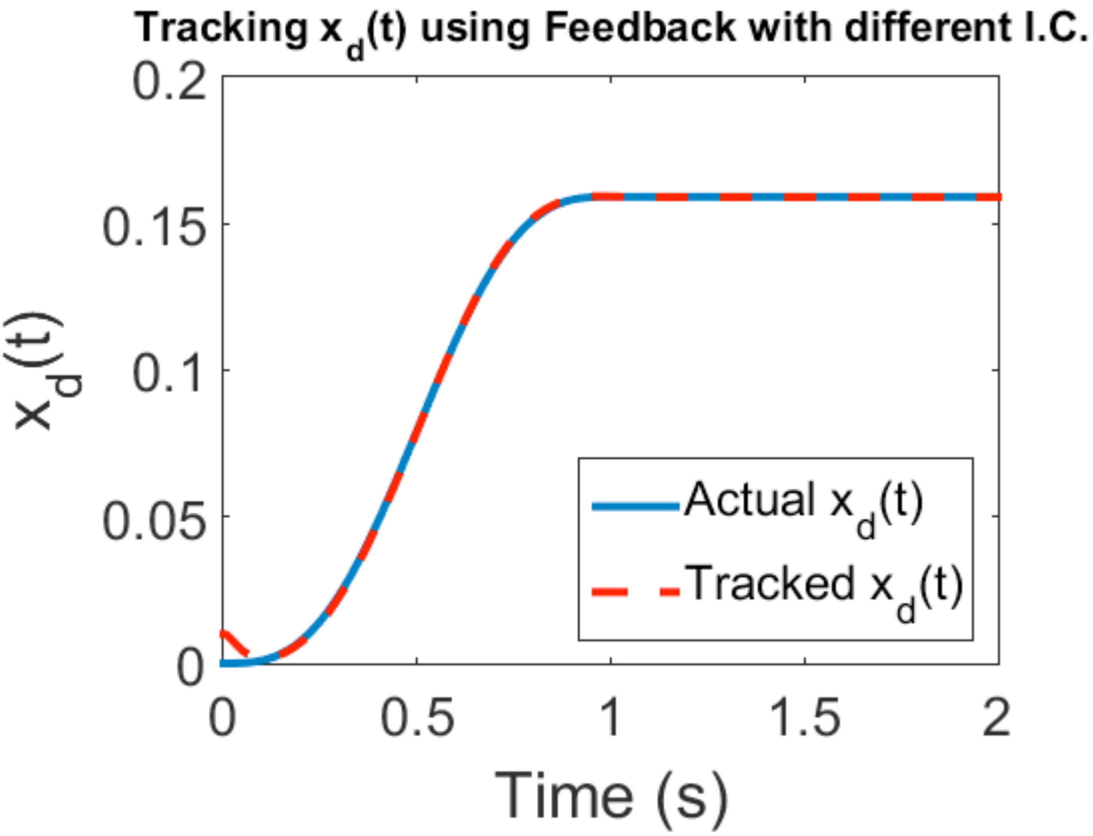
```
k_cl=place(A,B,[-35 -45]);
u_fb=(k_cl(1)*xd)+(k_cl(2)*vd);
A=[0 1 ; -k/M 0];
B=[0; 1/M];
C=[1 0];
D=0;
sys_cl=ss(A-B*k_cl,B,C,D);
y_cl=lsim(sys_cl,u_fb,t);
figure
plot(t,xd,'LineWidth',3)
hold on
plot(t,y_cl,'--r','LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
legend('Actual x_d(t)','Tracked x_d(t)','Location','southeast')
title('Tracking x_d(t) using Feedback with PD control','Fontsize',16)
```



Simulate the response of the system with feedback with different controller gain and I.C.

```
k_cl=place(A,B,[-35 -45]);
u_fb=(k_cl(1)*xd)+(k_cl(2)*vd);
A=[0 1 ; -k/M 0];
B=[0; 1/M];
C=[1 0];
D=0;
sys_cl=ss(A-B*k_cl,B,C,D);
y_cl=lsim(sys_cl,u_fb,t,x0);
figure
plot(t,xd,'LineWidth',3)
hold on
plot(t,y_cl,'--r','LineWidth',3)
xlabel('Time (s)','Fontsize',24)
ylabel('x_d(t)','Fontsize',24)
set(gca,'Fontsize',20)
legend('Actual x_d(t)','Tracked x_d(t)','Location','southeast')
title('Tracking x_d(t) using Feedback with different I.C.','Fontsize',16)
% %% comparing two input of iddferent control scheme
% k_cl1=place(A,B,[-35 -45]);
% u_fb1=(k_cl1(1)*xd)+(k_cl1(2)*vd);
% k_cl2=[0.1 0.1];
% %  k_cl=place(A,B,[-35 -45]);
% u_fb2=(k_cl2(1)*xd)+(k_cl2(2)*vd);
% figure
% plot(t,u_fb1,'LineWidth',3)
```

```
% hold on
% plot(t,u_fb2,'--r','LineWidth',3)
```



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