Preview-Based Stable-Inversion for Output Tracking of Linear Systems

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Stable Inversion techniques can be used to achieve high-accuracy output tracking. However, for nonminimum phase systems, the inverse is noncausal—hence the inverse has to be precomputed using a prespecified desired-output trajectory. This requirement for prespecification of the desired output restricts the use of inversion-based approaches to trajectory planning problems (for nonminimum phase systems). In the present article, it is shown that preview information of the desired output can be used to achieve online inversion-based output-tracking of linear systems. The amount of preview-time needed is quantified in terms of the tracking error and the internal dynamics of the system (zeros of the system). The methodology is applied to the online output tracking of a flexible structure and experimental results are presented.

1 Introduction

Inversion of system dynamics can be used to find inputs which achieve exact output-tracking (Hirschorn, 1979, Silverman, 1969). For systems with nonminimum phase dynamics, inputs found through standard inversion techniques tend to be unbounded and therefore, cannot to be used for practical output-tracking. Recently, stable inversion approaches (Devasia et al., 1992; Hunt et al., 1996; Bayo, 1987) have been developed for nonminimum phase systems, which yield bounded inputs for exact output-tracking. The application of such inversion-based output tracking has been studied for several nonminimum systems, like flexible manipulators (Paden et al., 1993; Kwon and Book, 1990), aircraft systems (Meyer et al., 1995; Tomlin et al., 1995; Devasia, 1997), and high precision positioning of piezo-probes for nano-technology (Croft et al., 1998). A critical difficulty with these inversion-based approaches is that the inverse is noncausal (for nonminimum phase systems) and therefore the desired output trajectory must be prespecified. This requirement for prespecification of the output-trajectory can be a substantial limitation on the use of the noncausal inversion-based approach, and it limits the inversion-based approach to trajectory planning applications. The main contribution of this paper is to show that, for linear systems, the noncausal inverse can be computed using a preview-based approach, which allows the inversion process to be applied online. This article also quantifies the amount of preview-time needed in terms of the desired tracking accuracy and the location of the zeros of the system. Implementation issues are discussed and the approach is experimentally verified by applying it to the output tracking of a flexible structure with nonminimum phase dynamics.

A major result in output tracking is the solution of the output regulation problem for linear systems by Francis (1977). These results were generalized for the nonlinear case by Isidori and Byrnes (1990). The desired outputs are assumed to be generated by an exosystem and the linear regulator is easily designed by solving a manageable set of linear equations. A problem, however, with the regulator approach is that the exosystem states are often switched to describe the desired output; this leads to transient tracking errors after the switching instants. Such switching-caused transient errors can be avoided by using inversion-based approaches to output tracking (Devasia et al., 1996; Devasia et al., 1997). Thus, it is advantageous to use inversion-based output-tracking when exact-tracking of a particular output trajectory is required. In the inversion based approach (Hirschorn, 1979; Silverman, 1969), the system dynamics is inverted to find the input that exactly tracks a single specified output trajectory (rather than track a class of outputs as in the case of the output-regulator). Inversion for nonminimum phase systems is challenging since the standard approaches lead to unbounded inputs (Hirschorn, 1979). Stable inversion techniques resolve this problem of unbounded inverse-inputs by finding bounded (but possibly noncausal) exact-tracking input-state trajectories (Devasia et al., 1996; Hunt et al., 1996). The noncausality of the inverse implies that the entire output trajectory needs to be known ahead of time which restricts the use of inversion-based approaches (for nonminimum phase systems). This motivates the current work, which shows that the noncausal inverse can be found by using preview information, and thereby, enables the online implementation of the inversion-based output-tracking technique for nonminimum phase systems. The focus of this article is to study continuous-time systems, however, the resulting methodology can be easily extended to the discrete-time case.

Other approaches have also used preview information of the desired output trajectory for output tracking, for example, to alleviate the problems due to nonminimum phase dynamics by exploiting actuator redundancy (Yin and Singh, 1997). Another use of previewed information of the output is in linear quadratic-based (lq-based) optimal output tracking controllers (see Lewis and Syrmos, 1995, Chapter 4). Tomizuka and coworkers (e.g., Tomizuka, 1993) have shown that the performance of finite-time-preview controllers approach the performance of the infinite-preview controller as the amount of preview time increases. In these works, the goal is to trade-off the tracking requirement to reduce the magnitude of the inputs. In contrast, inversion-based approaches aim to achieve high accuracy control of the desired output trajectory. Trading off the exact output-tracking requirement to achieve other goals like vibration minimization is also possible within the inversion-based framework (see Dewey et al., 1998). The resulting inverse controller is noncausal, which can also be implemented using the preview-based controller discussed in the current article.

The paper is organized in the following format. The inversion-based output-tracking scheme and its dependence on the solution of the system's internal dynamics is presented in Section 2. The inversion problem is then solved using preview information of the output, and implementation issues are studied in Section 3. In Section 4, the preview-based inversion approach is applied to the

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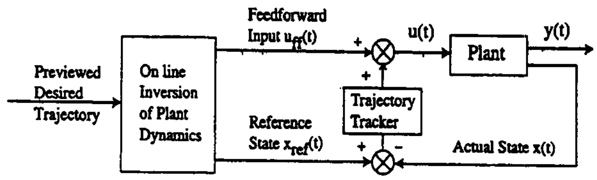

Fig. 1 The output tracking control scheme

output tracking control of a flexible structure and experimental results are presented. Discussions are in Section 5 and our conclusions are in Section 6.

2 Stable Inversion for Nonminimum Phase Systems

In this section, the inversion-based output tracking approach is presented. It is shown that solving the inversion problem is equivalent to finding bounded solutions to the system's internal dynamics.

2.1 Output Tracking Using Inversion of System Dynamics. Consider a linear system described by

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t), \tag{1}$$

which has the same number of inputs as outputs, $u(t), y(t) \in \Re^p$, and $x(t) \in \Re^n$. We assume that the system is stabilizable. Let $y_d(\cdot)$ be the desired output trajectory to be tracked. Then in the inversion-based approach we, first, find a nominal input-state trajectory, $[u_{ff}(\cdot), x_{ref}(\cdot)]$ that satisfies the system equations (1) and yields the desired output exactly, i.e.,

$$\left.\begin{array}{l}\dot{x}_{ref}(t) = Ax_{ref}(t) + Bu_{ff}(t)\\ y_d(t) = Cx_{ref}(t)\end{array}\right\} \quad \forall\, t \in (-\infty, \infty) \tag{2}$$

and, second, we stabilize the exact-output-yielding state trajectory, $x_{ref}(\cdot)$, by using state feedback (see Fig. 1). Thus $x(t) \to x_{ref}(t)$ and $y(t) \to y_d(t)$ as $t \to \infty$ and output tracking is achieved. It is noted that this stabilization can also be achieved by output feedback (rather than state feedback). While stabilization of the reference state trajectory can be easily achieved through standard techniques (Khalil, 1996) like state feedback of the form $K[x(t) - x_{ref}(t)]$, the main challenge is to find the inverse input-state trajectory $[u_{ff}(\cdot), x_{ref}(\cdot)]$—especially for systems with nonminimum phase dynamics. This paper addresses the on-line computation of the inverse input-state trajectory using preview information of the desired trajectory, y_d.

2.2 Stable Inversion Scheme. In this subsection, it is shown that finding the inverse input-state trajectory is equivalent to finding bounded solutions to the system's internal dynamics. Let the linear system (1) have a well-defined vector relative degree, $r := [r_1, r_2, \ldots, r_p]$. Then the output's derivatives are given as:

$$\frac{d^{r_k}y_k}{dt^{r_k}} = C_kA^{r_k}x + C_kA^{r_k-1}Bu \tag{3}$$

where C_k is the k^{th} row of C, and $1 \le k \le p$. In vector notation let Eq. (3) be rewritten as

$$y^{(r)}(t) = A_xx(t) + B_yu(t) \tag{4}$$

where

$$y^{(r)} := \left[\frac{d^{r_1}y_1}{dt^{r_1}}\,\frac{d^{r_2}y_2}{dt^{r_2}}\cdots\frac{d^{r_p}y_p}{dt^{r_p}}\right]^T$$

$$A_x := \begin{bmatrix} C_1A^{r_1}\\ C_2A^{r_2}\\ \vdots\\ C_pA^{r_p}\end{bmatrix}; \quad B_y := \begin{bmatrix} C_1A^{r_1-1}B\\ C_2A^{r_2-1}B\\ \vdots\\ C_pA^{r_p-1}B\end{bmatrix},$$

and B_y is invertible because of the well-defined relative degree assumption. Equation (4) motivates the choice of the control law of the form

$$u_{ff}(t) = B_y^{-1}[y_d^{(r)}(t) - A_xx(t)] \tag{5}$$

for all $t \in (-\infty, \infty)$. Substituting this control law in Eq. (4), it is seen that exact tracking is maintained, i.e.,

$$y^{(r)}(t) = y_d^{(r)}(t).$$

To study the effect of this control law, consider a change of coordinates T such that

$$\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} = Tx(t) \tag{6}$$

where $\xi(t)$ consists of the output and its time-derivatives

$$\xi(t) := \left[y_1, \dot{y}_1, \ldots, \frac{d^{r_1-1}y_1}{dt^{r_1-1}},\ y_2, \dot{y}_2, \ldots, \frac{d^{r_2-1}y_2}{dt^{r_2-1}}, \ldots,\ y_p, \dot{y}_p, \ldots, \frac{d^{r_p-1}y_p}{dt^{r_p-1}}\right]'.$$

The system equation (1) can then be rewritten in the new coordinates as

$$\dot{\xi}(t) = \hat{A}_1\xi + \hat{A}_2\eta + \hat{B}_1u \tag{7}$$

$$\dot{\eta}(t) = \hat{A}_3\xi + \hat{A}_4\eta + \hat{B}_2u \tag{8}$$

where

$$\hat{A} := \begin{bmatrix}\hat{A}_1 & \hat{A}_2\\ \hat{A}_3 & \hat{A}_4\end{bmatrix} := TAT^{-1}; \quad \text{and} \quad \hat{B} := \begin{bmatrix}\hat{B}_1\\ \hat{B}_2\end{bmatrix} = TB$$

In the new coordinates, the control law for maintaining exact tracking (Eq. (5)) can be written as

$$u_{ff}(t) = B_y^{-1}[y_d^{(r)}(t) - A_\xi\xi_d(t) - A_\eta\eta(t)] \tag{9}$$

where

$$[A_\xi \quad A_\eta] := A_xT^{-1}.$$

Note that the desired $\xi(\cdot)$ is known when the desired output trajectory $y_d(\cdot)$ and the output's time derivatives are specified. This desired $\xi(\cdot)$ is defined as $\xi_d(\cdot)$. Since the control law was chosen such that exact tracking is maintained, $y^{(r)}(t) = y_d^{(r)}(t)$ we also have $\xi(t) = \xi_d(t)$, and Eqs. (7) and (8) become

$$\xi(t) = \xi_d(t) \tag{10}$$

$$\dot{\eta}(t) = \hat{A}_3\xi_d(t) + \hat{A}_4\eta(t) + \hat{B}_2B_y^{-1}[y_d^{(r)}(t) - A_\xi\xi_d(t) - A_\eta\eta(t)]$$
$$:= \hat{A}_\eta\eta(t) + \hat{B}_\eta\mathbb{Y}_d(t) \tag{11}$$

where

$$\hat{A}_\eta := \hat{A}_4 - \hat{B}_2B_y^{-1}A_\eta; \tag{12}$$

$$\hat{B}_\eta := [\hat{B}_2B_y^{-1} \quad (\hat{A}_3 - \hat{B}_2B_y^{-1}A_\xi)]; \quad \text{and} \quad \mathbb{Y}_d := \begin{bmatrix} y_d^r(t)\\ \xi_d(t)\end{bmatrix}$$

This is the inverse system, and in particular, Eq. (11) represents the internal dynamics. If a bounded solution, $\eta_d(\cdot)$, to the internal dynamics (11) can be found then the exact-tracking feedforward input can be found through Eq. (9) as

$$u_{ff}(t) = B_y^{-1}[y_d^{(r)}(t) - A_\xi\xi_d(t) - A_\eta\eta_d(t)] \tag{13}$$

and the reference state trajectory can be found from Eq. (6) as

$$x_{ref}(t) = T^{-1}\begin{bmatrix}\xi_d(t)\\ \eta_d(t)\end{bmatrix}. \tag{14}$$

Thus, a bounded solution to the internal dynamics (11) is required to find the inverse for applying the output tracking scheme shown in Fig. 1.

2.3 Bounded Solutions to the Internal Dynamics. We restrict the following discussion to systems with hyperbolic internal dynamics, i.e., none of the zeros of the system (Eq. (1)) lie on the imaginary axis of the complex plane (a technique to address the nonhyperbolic case can be found in Devasia, 1997). If the internal dynamics is hyperbolic, there exists a transformation U such that the internal dynamics (11) can be decoupled into a stable subsystem (σ_s) and an unstable subsystem (σ_u):

$$\dot{\sigma}_s(t) = \tilde{A}_s\sigma_s(t) + \tilde{B}_s\mathbb{Y}_d(t) \quad (15)$$

$$\dot{\sigma}_u(t) = \tilde{A}_u\sigma_u(t) + \tilde{B}_u\mathbb{Y}_d(t) \quad (16)$$

where

$$\sigma(t) := \begin{bmatrix} \sigma_s(t) \\ \sigma_u(t) \end{bmatrix} = U\eta(t) \quad (17)$$

Bounded solution to the internal dynamics in the transformed coordinates can then be found as

$$\sigma_s(t) = \int_{-\infty}^{t} e^{\tilde{A}_s(t-\tau)}\tilde{B}_s\mathbb{Y}_d(\tau)d\tau \quad (18)$$

$$\sigma_u(t) = -\int_{t}^{\infty} e^{-\tilde{A}_u(\tau-t)}\tilde{B}_u\mathbb{Y}_d(\tau)d\tau \quad (19)$$

In the new coordinates, the feedforward control law in Eq. (13) can be written as:

$$u_{ff}(t) = B_y^{-1}[y_d^r(t) - A_\xi\xi_d(t) - A_\eta\hat{U}_s\sigma_s(t) - A_\eta\hat{U}_u\sigma_u(t)] \quad (20)$$

where $U^{-1} := [\hat{U}_s \;\; \hat{U}_u]$ is partitioned according to the partition of σ in Eq. (17).

This completes the inversion technique. To summerize: the bounded solutions found through Eqs. (18) and (19) are used to find a bounded solution to the internal dynamics, η_d, by using Eq. (17). The inversion is then completed by finding the reference state and input trajectories by using Eqs. (14) and (20), which are then used in the control scheme shown in Fig. 1 to obtain output tracking. Note that only the past information is needed to compute the solution to the stable subsystem of the internal dynamics by using Eq. (18). However, to find a bounded solution to the unstable subsystem (σ_u) by using Eq. (19), the desired output must be completely specified (including future information). This is the main problem, which restricts the use of inversion to trajectory planning (where it may be acceptable to solve Eq. (19) off-line).

3 Preview Based Inversion

In this section, we discuss the online computation and implementation of the inverse using preview-information of the desired output trajectory, which enables the tracking of output trajectories that are specified on-line. We begin by quantifying the relationship between the preview-time and tracking error.

Let the desired output, y_d (and its time-derivatives) be given for a preview-time of T_p seconds, i.e., at time t, $\mathbb{Y}_d(\tau)$ (defined in Eq. (12)) is known for all $t \le \tau \le t + T_p$. This preview information is used to approximate the solution to the internal dynamics (in particular, to approximate the bounded solution to the unstable subsystem, σ_u, given by Eq. (19)). The approximated solution, $\tilde{\sigma}_u$ is found as

$$\tilde{\sigma}_u(t) = -\int_{t}^{t+T_p} e^{-\tilde{A}_u(\tau-t)}\tilde{B}_u\mathbb{Y}_d(\tau)d\tau \quad (21)$$

Let the error between the exact-solution to the unstable subsystem found through Eq. (19) and the approximated-solution found through Eq. (21) be defined as:

$$e_{\sigma_u} = \sigma_u(t) - \tilde{\sigma}_u(t) \quad (22)$$

We show in the following Lemma that $e_{\sigma_u}(t)$ can be made arbitrarily small by having a large enough preview time. Furthermore, we show that the error in computation of the inverse input by using the finite preview also converges to zero as the preview time, T_p, increases. The error in computing the inverse-input, $e_u(t)$, is defined as

$$e_u(t) = \tilde{u}_{ff}(t) - u_{ff}(t) \quad (23)$$

where $u_{ff}(t)$ denotes the input obtained with infinite preview using Eq. (20), and $\tilde{u}_{ff}(t)$ denotes the preview input obtained with finite preview when σ_u is replaced by the approximate solution $\tilde{\sigma}_u$ in Eq. (20)

$$\tilde{u}_{ff}(t) := B_y^{-1}[y_d^r(t) - A_\xi\xi_d(t) - A_\eta\hat{U}_s\sigma_s(t) - A_\eta\hat{U}_u\tilde{\sigma}_u(t)]. \quad (24)$$

This implies that, as the preview time increases, the approximated input trajectory approaches (arbitrarily closely) the exact output-tracking inverse trajectory.

Lemma 1. Let the desired trajectory and its time derivatives be bounded, i.e. there exists a positive scalar $M \in \mathbb{R}$, such that $\|\mathbb{Y}_d(t)\|_2 \le M$ for all time t. Then for any scalar $\epsilon > 0$, there exists a finite time T_p^* such that the error (in computing the inverse input by using the preview information of the output) can be made smaller than ϵ if the preview time is larger than T_p^*, i.e., $\|e_u(t)\|_2 \le \epsilon$. In the above notation, $\|z\|_2$ is the standard Euclidean norm for any vector z.

Proof: Since the subsystem (16) is unstable, $-\tilde{A}_u$ is Hurwitz, and therefore positive scalars α and κ can be chosen such that (Desoer and Vidyasagar, 1975)

$$\|e^{-\tilde{A}_u t}\|_2 \le \kappa e^{-\alpha t} \quad \forall\, t > 0 \quad (25)$$

where, given a matrix $F \in \mathbb{R}^{n\times n}$, $\|F\|_2$ denotes the induced matrix norm, defined as $\|F\|_2 := \sup_{\|x\|_2=1}\|Fx\|_2$. From the definition of e_{σ_u} in Eq. (22), we obtain

$$\|e_{\sigma_u}(t)\|_2 = \|\sigma_u(t) - \tilde{\sigma}_u(t)\|_2$$

$$= \left\|\int_{t+T_p}^{\infty} e^{-\tilde{A}_u(\tau-t)}\tilde{B}_u\mathbb{Y}_d(\tau)d\tau\right\|_2$$

(using Equations (19) and (21))

$$\le \int_{t+T_p}^{\infty} \|e^{-\tilde{A}_u(\tau-t)}\|_2\|\tilde{B}_u\mathbb{Y}_d(\tau)\|_2 d\tau$$

$$\le K_1\int_{t+T_p}^{\infty} \|e^{-\tilde{A}(\tau-t)}\|_2 d\tau \quad (\text{with } K_1 := M\|\tilde{B}_u\|_2)$$

$$\le \frac{K_1\kappa}{\alpha}e^{-\alpha T_p} \quad (\text{using Eq. (25)}) \quad (26)$$

From the definition of the computational-error in finding the inverse-input (Eq. (23)) and from Eqs. (20), (22), and (24), we have:

$$\sup_t \|e_u(t)\|_2 \le \sup_t \|B_y^{-1}\|_2 \|A_\eta\|_2 \|\hat{U}_u\|_2 \|e_{\sigma_u}(t)\|_2$$

$$:= \sup_t K_2 \|e_{\sigma_u}(t)\|_2 \quad (K_2 := \|B_y^{-1}\|_2 \|A_\eta\|_2 \|\hat{U}_u\|_2)$$

$$\le \frac{\mathcal{K}}{\alpha} e^{-\alpha T_p},$$

$$\text{(using inequality (26) and } \mathcal{K} := K_1 K_2 \kappa) \qquad (27)$$

For any given $\epsilon > 0$, choosing

$$T_p^* \ge \frac{\ln \dfrac{\mathcal{K}}{\epsilon\alpha}}{\alpha}. \qquad (28)$$

and substituting any $T_p > T_p^*$ in Eq. (27) completes the proof. □

It is noted that for proving this lemma, it is sufficient for $\|\tilde{B}_u \Upsilon_d(\cdot)\|_2$ to be bounded, which can be less restrictive than requiring $\|\Upsilon_d(\cdot)\|_2$ to be bounded. Using similar arguments, it is also possible to show that the error in computing the inverse reference-state-trajectory, x_{ref}, can be made arbitrarily small by choosing a sufficiently large preview time.

The next lemma shows that the error in output-tracking, due to errors in computing the inverse, can also be made small by choosing a sufficiently large preview time (for a similar argument, see Devasia and Paden, 1998)

Lemma 2. Let the original system (1) be stable (or stabilized with feedback before the inversion is applied). Then the tracking error, on applying the finite-preview-based input ($\tilde{u}_{ff}$) as a feedforward input, can be made arbitrarily small by choosing a large enough preview time.

Proof: From Eq. (1), the dynamics of the state-error $e_x(t) := x(t) - x_{ref}(t)$, where $x(t)$ is the system-state when the finite preview input $\tilde{u}_{ff}(t)$ is applied to system (1), can be described by

$$\dot{e}_x(t) = A e_x(t) + B e_u(t) \qquad (29)$$

The error in output-tracking $e_y(t) := y(t) - y_d(t) = C e_x(t)$ can then be bounded as

$$\|e_y(t)\|_2 = \|C e_x(t)\|_2 \le \|C\|_2 \|e_x(t)\|_2$$

$$\le \|C\|_2 \int_{-\infty}^{t} \|e^{A(t-\tau)}\|_2 \|B\|_2 \|e_u(\tau)\|_2 d\tau$$

$$\text{(using solution of Equation (29))}$$

$$\le \|C\|_2 \|B\|_2 (\sup_\tau \|e_u(\tau)\|_2) \int_{-\infty}^{t} \|e^{A(t-\tau)}\|_2 d\tau \qquad (30)$$

If the original system (Eq. (1)) is stable (or if the inversion is carried out after stabilizing the system), then there exist real positive numbers M_1 and β such that $\|e^{A(t-\tau)}\|_2 \le M_1 e^{-\beta(t-\tau)}$. Substituting this in Eq. (30) yields

$$\|e_y(t)\|_2 \le \|C\|_2 \|B\|_2 (\sup_\tau \|e_u(\tau)\|_2) \int_{-\infty}^{t} M_1 e^{-\beta(t-\tau)} d\tau$$

$$= M_2 \left(\frac{M_1}{\beta}\right) (\sup_\tau \|e_u(\tau)\|_2) \quad (\text{with } M_2 := \|C\|_2 \|B\|_2)$$

$$\le \frac{M_1 M_2 \mathcal{K}}{\alpha\beta} e^{-\alpha T_p} \quad \text{(using inequality (27))} \qquad (31)$$

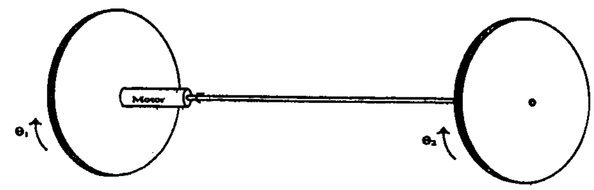

Fig. 2 The experimental flexible structure

The tracking error, $\|e_y(t)\|_2$, can be kept smaller than any given $\epsilon_y > 0$ for all time, t, by choosing the preview time, T_p, as

$$T_p \ge T_p^* \ge \frac{\ln \dfrac{M_1 M_2 \mathcal{K}}{\alpha\beta\epsilon_y}}{\alpha} \qquad (32)$$

which completes the proof. □

Remark. The preview time T_p needed to achieve a desired accuracy in output tracking depends on the distance of the right half plane zeros of the system (Eq. (1)) from the imaginary axis of the complex plane. As this distance increases, α in Eq. (32) can be chosen larger (defined in Eq. (25)) and therefore, a smaller preview-time can be chosen to achieve the desired accuracy in output-tracking.

4 Example: Flexible Structure Control

The experimental flexible structure considered here consists of two discs which are connected by a thin freely rotating shaft as shown in Fig. 2. The system input, $U(t)$, is the voltage (Volts) applied to a DC motor, and the output, θ_2, is the angular rotation (in degrees) of the disc which is farther from the motor. The control objective is to make the output (angular rotation, θ_2) track a desired output-trajectory, which is specified on-line. The system equations (obtained experimentally with a HP3562A Dynamic Signal Analyzer) can be written in the following state-space form where the state vector, x is chosen as

$$x := [\theta_1 \ \dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2]^T$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3.656 & -0.436 & 3.573 & -0.091 \\ 0 & 0 & 0 & 1 \\ 3.245 & -0.126 & -3.259 & -0.076 \end{bmatrix} x + \begin{bmatrix} 0 \\ 21.9027 \\ 0 \\ 3.588 \end{bmatrix} u \qquad (33)$$

$$:= Ax + Bu$$

$$y := \theta_2 = [0 \ 0 \ 1 \ 0] x \qquad (34)$$

More details of the flexible structure can be found in Dewey et al. (1998).

4.1 System Inverse. The relative degree of the above system is two and hence the output has to be differentiated twice to relate the input and the output (as described in Section (2.2) with $\xi := [y, \dot{y}]^T = [\theta_2, \dot{\theta}_2]^T$). Note that ξ is known when preview information of the desired output and its derivatives are defined. The internal dynamics of the system are described by $\eta := [\theta_1, \dot{\theta}_1]^T$. The inverse input u_{ff} can be written as (using Eq. (13) and the last row of Eq. (33))

$$u_{ff}(t) = \frac{1}{3.588} \{\ddot{y}_d(t) - [3.245 - 0.126]\eta(t) - [-3.259 - 0.076]\xi_d(t)\} \qquad (35)$$

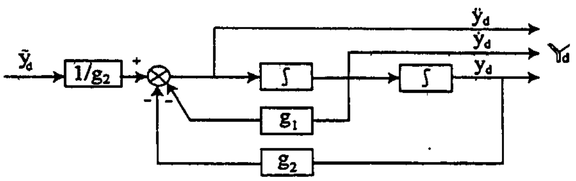

Fig. 3 The state-variable filter used to generate the desired trajectory, (y_d), and its time derivatives (Irvine, 1981; Stanley, 1985)

and the internal dynamics is given by substituting this control law into the first two rows of Eq. (33)

$$\dot{\eta} = \begin{bmatrix} 0 & 1 \\ -23.4658 & 0.3306 \end{bmatrix} \eta + \begin{bmatrix} 0 & 0 & 0 \\ 23.4658 & 0.3753 & 6.1041 \end{bmatrix} \mathbb{Y}_d \quad (36)$$

$$:= A_\eta \eta + B_\eta \mathbb{Y}_d \quad (37)$$

where, $\mathbb{Y}_d = [y_d \ \dot{y}_d \ \ddot{y}_d]^T$ and the entire internal dynamics is unstable, i.e., $\sigma_u = \eta$.

4.2 $\mathbb{Y}_d$ Generation. There are several methods available to generate the desired trajectory y_d for a preview time of T_p. Note that to compute the inversion-based control, the time-derivatives of the desired output trajectory must also be specified. One method to generate the previewed desired output trajectory is to predict the future output by using polynomial extrapolations of the past desired trajectory signals (Miller and Pachter, 1997) and then to differentiate these polynomials. Another approach is to use an exosystem as a command generator (Tomizuka, 1975) and then to switch its states to generate online changes of the desired trajectory. For our experimental system, a potentiometer was adjusted to define the desired output, $\tilde{y}_d$. Next, the angular-velocity and angular-acceleration of the desired output was obtained by successive time-differentiation of the desired output signal, $\tilde{y}_d$. To avoid noise problems associated with direct differentiation, we used a state-variable filter (Irvine, 1981; Stanley, 1985), shown in Fig. 3, to generate a filtered output trajectory, y_d, and its time-derivatives $\dot{y}_d$, $\ddot{y}_d$. The cut-off frequency of the state-variable filter was set to 1 Hz to match the modeled bandwidth of the flexible structure—higher-frequency dynamics are not represented by Eq. (33). This signal y_d, obtained by filtering the command signal ($\tilde{y}_d$), was considered to be the previewed desired-output, that must be achieved by the system after the preview time T_p. Thus, the command-signal $\tilde{y}_d(t)$ generated by the user (by adjusting a potentiometer) is considered as the previewed signal; it is filtered and differentiated to generate the preview of the desired output trajectory and its time-derivatives ($\mathbb{Y}_d$). This scheme to generate the desired output trajectory (with preview) and its use in the inversion scheme is shown in Fig. 4.

4.3 Online Implementation of Preview-Based Inversion. The preview-based solution to the unstable subsystem of the internal dynamics, i.e., the integral Eq. (21), was computed online, which can be done by numerical integration, for example, by using the fourth-order Simpson formula (Kincaid and Cheney, 1991) which approximates a continuous integral as follows:

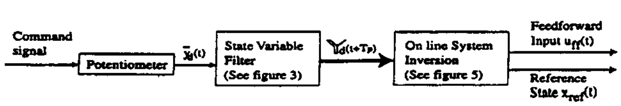

Fig. 4 The preview-based inversion scheme with online trajectory-generation

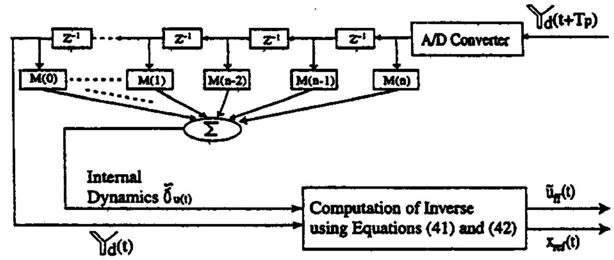

Fig. 5 The schematic of on-line calculation of the system inversion

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(x_0) + 2\sum_{i=2}^{n/2} f(x_{2i-2}) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + f(x_n)\right] \quad (38)$$

where n is even and $h = (b - a)/n$. Using this numerical integration method, the solution to the internal dynamics (21) was approximated as:

$$\tilde{\sigma}_u(t) = -\int_t^{t+T_p} e^{-\tilde{A}_u(\tau - t)}\tilde{B}_u \mathbb{Y}_d(\tau)d\tau \approx \frac{h}{3}\sum_{i=0}^{N} \mathcal{M}(i)\mathbb{Y}_d(t, i) \quad (39)$$

where the preview time (T_p) is an even-integer multiple of the sampling time (T_s), i.e., $N := T_p/T_s$ is an even integer, $\mathbb{Y}_d(t, i) := \mathbb{Y}_d(t + i*T_s)$ and:

$$\mathcal{M}(i) = \begin{cases} -e^{-\tilde{A}_u}\tilde{B}_u & \text{if } i = 0 \\ -2*e^{-\tilde{A}_u(i*T_s)}\tilde{B}_u & \text{if } i > 0 \text{ and } i \text{ is even} \\ -4*e^{-\tilde{A}_u(i*T_s)}\tilde{B}_u & \text{if } i \text{ is odd} \\ -e^{-\tilde{A}_u(n*T_s)}\tilde{B}_u & \text{if } i = n \end{cases} \quad (40)$$

Note that the matrices $\mathcal{M}(i)$ can be precomputed and stored to reduce the online computations. This computation-scheme is shown in Fig. 5. The computation-error, due to time-discretization, can be made negligible by choosing a small enough sampling time (T_s). Once a bounded solution $\tilde{\sigma}_u$ to the unstable part of the internal dynamics is found, the inverse-input is computed by using Eq. (35)

$$u_{ff}(t) = \frac{1}{3.588}\{\ddot{y}_d(t) - [3.245 - 0.126]\tilde{\sigma}_u(t) - [-3.259 - 0.076]\xi_d(t)\}, \quad (41)$$

and the reference state trajectory is found as

$$x_{ref} = \begin{bmatrix} \tilde{\sigma}_u \\ \xi_d \end{bmatrix} \quad (42)$$

4.4 Experimental Results. Experimental results for output-tracking with two-different preview-times, $T_p = 20$ seconds and $T_p = 50$ seconds, are shown in Fig. 6, which illustrate the improvements in output-tracking as the preview-time is increased. The experiments include feedback-stabilization, which was added to account for unmodeled dynamics in the system (like friction in the experimental system and static imbalance of the discs, which created a tendency in the disks to settle in a specific orientation). It is noted that desired output trajectories for the two cases are different (shown as dashed lines in Fig. 6) because these desired output trajectories were generated on-line for the two different

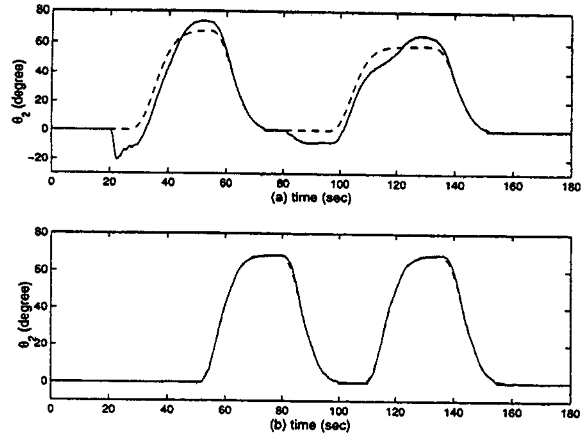

Fig. 6 Experimental results. The dashed line is the desired trajectory, the solid line represents the output trajectory. The top plot is for 20 seconds of preview time, and the bottom plot is for 50 seconds of preview time.

preview-time cases. As shown in Fig. 6, preview-based inversion improves tracking performance with increasing preview time, and online specification of the desired output trajectory is possible for noncausal inversion.

5 Discussion

For nonminimum phase systems, recent stable inversion-based approaches can be used to achieve high-accuracy output tracking. However, the entire desired output-trajectory has to be prescribed for the off-line computation of the inverse—this is a significant limitation since the desired trajectory cannot be changed online. This limitation has been alleviated by the current preview-based approach which allows the online implementation of inversion-based output-tracking controllers. The methodology allows the application of the inversion-based approach to systems like flexible manipulators and servo-positioning systems where online-changes of the desired trajectory may be necessary. Such online-changes are allowed by the current technique if a preview of the desired trajectory is possible. This requirement for preview information is not a drawback of the current controller. Rather, this is necessary for high-accuracy output tracking because there are performance limitations on output-tracking for nonminimum phase systems if output-preview is not available (Qiu and Davison, 1993). An important result of the preview-based inversion approach is that the output-tracking error can be made arbitrarily small by choosing a sufficiently large preview time. Further, the quantification of the amount of preview time in terms of the location of system-zeros in the imaginary plane (see Section 3) can aid in the initial design of a system. In particular, the preview time needed can be made smaller by choosing design-parameters such that right-half plane zeros (if any) are far away from the imaginary axis of the complex plane. This preview-based stable-inversion can also be extended to the discrete-time case. In summary, the preview-based approach will help in the implementation of inversion-based control laws for high-accuracy, on-line output-tracking of nonminimum phase systems.

6 Conclusion

We have developed a preview-based output-tracking controller using on-line inversion for nonminimum phase systems. The preview time needed was quantified in terms of the required accuracy in output tracking, and related to the system-zeros. Implementation issues were discussed and the technique was illustrated by applying it for output-tracking of a flexible structure experiment.

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References

Bayo, E., 1987, "A Finite-Element Approach to Control the End-Point Motion of a Single-Link Flexible Robot," *Journal of Robotic Systems*, Vol. 4(1), pp. 63–75.

Croft, D., Stilson, S., and Devasia, S., 1999, "Optimal Tracking of Piezo-Based Nano-Positioners," *Nanotechnology*, Vol. 10, June. pp. 201–208.

Desoer, C., and Vidyasagar, M., 1975, *Feedback Systems: Input-Output Properties*, Academic Press, N.Y., Chapter 2.

Devasia, S., and Paden, B., 1998, "Stable Inversion for Nonlinear Nonminimum-Phase Time-Varying Systems," *IEEE Transactions on Automatic Control*, Vol. 43(2), Feb., pp. 283–288.

Devasia, S., 1997, "Output Tracking with Nonhyperbolic and Near Nonhyperbolic Internal Dynamics: Helicopter Hover Control," *Journal of Guidance, Control, and Dynamics*, Vol. 20(3), pp. 573–580.

Devasia, S., Chen, D., and Paden, B., 1996, "Nonlinear Inversion-Based Output Tracking," *IEEE Transactions on Automatic Control*, Vol. 41(7), pp. 930–943.

Devasia, S., Paden, B., and Rossi, C., 1997, "Exact-Output Tracking Theory for Systems with Parameter Jumps," *International Journal of Control*, Vol. 67(1), pp. 117–131.

Dewey, J., Leang, K., and Devasia, S., 1998, "Experimental and Theoretical Results in Output-Trajectory Redesign for Flexible Structures," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 120, pp. 456–461.

Francis, B. A., 1977, "The Linear Multi-Variable Regulator Problem," *Journal of Control Optimization*, Vol. 15, pp. 486–505.

Hirschorn, R. M., 1979, "Invertibility of Multivariable Nonlinear Control System," *IEEE Transactions on Automatic Control*, Vol. 24(6), pp. 855–865.

Hunt, L. R., Meyer, G., and Su, R., 1996, "Noncausal Inverses for Linear Systems," *IEEE Transactions on Automatic Control*, Vol. 41(4), pp. 608–611.

Irvine, G. R., 1981, *Operational Amplifier Characteristics and Applications*, Prentice-Hall, Englewood Cliffs, NJ, Chapter 9.

Isidori, A., and Byrnes, C. I., 1990, "Output Regulation of Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 35(2), pp. 131–140.

Khalil, H. K., 1996, *Nonlinear Systems*, Prentice-Hall, Second Edition.

Kincaid, D., and Cheney, W., 1991, *Numerical Analysis*, Brooks/Cole, Pacific Grove, CA, Chapter 7.

Kwon, D., and Book, W. J., 1990, "An Inverse Dynamic Method Yielding Flexible Manipulator State Trajectories," *Proceedings of the American Control Conference*.

Lewis, F. L., and Syrmos, V. L., 1995, *Optimal Control*, John Wiley, New York, Second Edition.

Meyer, G., Hunt, L. R., and Su, R., Jan. 1995, "Nonlinear System Guidance in the Presence of Transmission Zero Dynamics," NASA Technical Memorandum (4661).

Miller, R. B., and Pachter, M., 1997, "Maneuvering Flight Control with Actuator Constraints," *Journal of Guidance, Control, and Dynamics*, Vol. 20(4), pp. 729–734.

Paden, B., Chen, D., Ledesma, R., and Bayo, E., 1993, "Exponentially Stable Tracking Control for Multi-Joint Flexible Manipulators," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 115(1), pp. 53–59.

Qiu, L., and Davison, E. J., 1993, "Performance Limitations of Non-Minimum Phase Systems in the Servomechanism Problem," *Automatica*, Vol. 29, Mar., pp. 337–349.

Silverman, L. M., 1969, "Inversion of Multivariable Linear Systems," *IEEE Transactions on Automatic Control*, Vol. 14(3), pp. 370–376.

Stanley, D. W., 1985, *Operational Amplifiers with Linear Integrated Circuits*, Charles E. Merrill, Columbus, Ohio, Chapter 7.

Tomizuka, M., 1975, "Optimal Continuous Finite Preview Problem," *IEEE Transactions on Automatic Control*, June, pp. 362–365.

Tomizuka, M., 1993, "On the Design of Digital Tracking Controllers," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 115, June, pp. 412–418.

Tomlin, C., Lygeros, J., and Sastry, S., 1995, "Output Tracking for a Nonminimum Phase Dynamic CTOL Aircraft Model," *IEEE Proceedings of the Conference on Decision and Control*, pp. 1867–1872.

Yin, W., and Singh, S. N., 1997, "Nonlinear Inverse and Predictive End Point Trajectory Control of Flexible Macro-Micro Manipulators," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 119(3), Sept., pp. 412–420.