## Washington Experimental Mathematics Lab Brownian Bridges

Department of Mathematics University of Washington

Autumn 2017

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## Introduction to the Theory of Brownian Bridges

Definition An *m*-dimensional Wiener (or Brownian Motion) process with mean  $\mu$  and variance  $\sigma^2$  is a stochastic process  $(W_t)_{t>0}$  with state space  $\mathbb{R}^m$  satisfying:

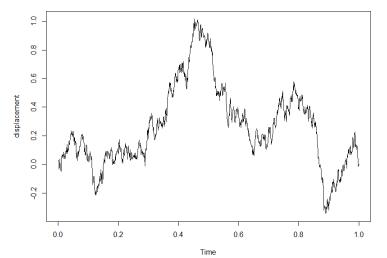
Remark Let  $(W_t)_{t\geq 0}$  be an *m*-dimensional Wiener process with mean  $\mu$ , variance  $\sigma^2$ , and let T > 0. Define  $(B_t)_{t\in[0,T]}$  by

$$B_t \equiv W_t - rac{t}{T} W_T.$$

 $(B_t)_{t \in [0,T]}$  is called a Brownian bridge on [0, T] with parameters  $\mu, \sigma^2$ .

## **Brownian Motion**



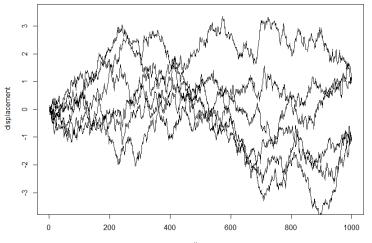


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## 1D Randomized End Points

### Randomized Bb with end points of 1 or -1



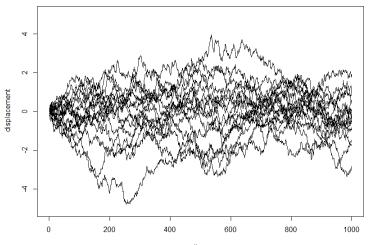
time

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## 1D End Points From Random Normal

#### Randomized Bb with end points from N(0,1)



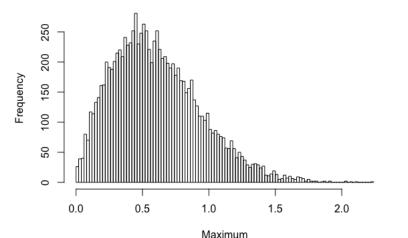
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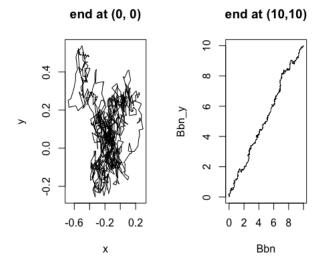
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## 1D Distribution of Maximum Values

### Maximums of Brownian Bridges with end points of 0



## 2D Brownian Bridges Fixed Start End Point

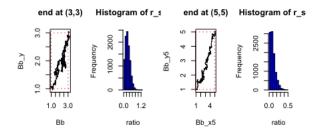


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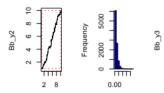
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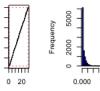
## 2D Brownian Bridges different End Point AND Probability of exist



end at (10,10) Histogram of r\_s







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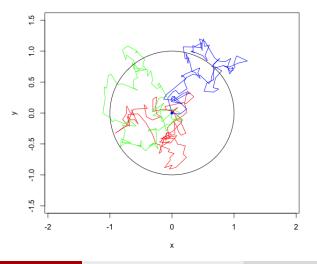
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## 2D Brownian Bridges Example



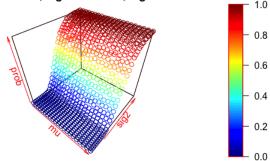
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# 2D Brownian Bridges Exit Probabilities from the Open Unit Disk

Brownian bridge (on [0,1]) exit probability of exiting the open disk of radius: 1 mu-start: -1 , mu-end: 1 , mu-incr: 0.1 sig2-start: 0.01 , sig2-end: 0.2 , sig2-incr: 0.005



# 2D Brownian Bridges First Exit Times Distributions Evolution

link

https://goo.gl/XV5kMq

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## Sources

- https://www.math.ucdavis.edu/ hunter/m280\_09/ch5.pdf
- https://en.wikipedia.org/wiki/Wiener\_process
- https://en.wikipedia.org/wiki/Brownian\_bridge
- http://www.columbia.edu/ ks20/FE-Notes/4700-07-Notes-E