Longest Orbits over Varieties of Generalized Markoff Equations over Finite Fields

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Abstract

The automorphism group of the polynomial $k(x, y, z) = x^2+y^2+z^2-xyz-2$ over a finite field $\mathbb{F}_p^3$ has a subgroup Γ, consisting of polynomal automorphisms, whose orbit lengths are of particular fascination. The group Γ is generated by the automorphisms ι, τ, η and acts on the variety $\mathbb{V}(k-\lambda)$, for λ in $\mathbb{F}$. We are interested in the length of the longest orbit, denoted $\mathcal{L}_{\langle w\rangle}(p, \lambda)$, for a fixed λ and prime p, where $\langle w\rangle$ is the cyclic subgroup generated by w in Γ. The evaluation of our $\mathcal{L}$ function is complete for ι, τ, $\iota\tau$, and $\eta\iota$.

A motivating conjecture is that these automorphisms will act transitively on the variety as p tends toward infinity. We hope to gain insight into how the gorup action is approaching transitivity by studying automorphisms of exceptionally large order. The automorphism $\eta\tau$ is of interest due to its orbits rate of growth as p increases; It appears to tend toward $p \log p$. This is significantly larger than the linear growth rate of η, or the constant orders of ι, τ, $\iota\tau$, and $\eta\iota$.